

# Image blending



15-463, 15-663, 15-862  
Computational Photography  
Fall 2017, Lecture 7

# Course announcements

- September 27<sup>th</sup> lecture tentatively rescheduled for  
Friday 29<sup>th</sup>, 12:00-1:30pm (same time, different day)
  - Still looking for room.
  - Will announce on Piazza and update course website once room is confirmed.
- Homework 1 scores have been uploaded on Canvas.
  - Mean score: 102.
  - Median score: 100.
- If you haven't started Homework 2 yet, you should.

# Overview of today's lecture

- Some motivating examples.
- Cut-and-paste.
- Alpha (linear) blending.
- Multi-band blending.
- Poisson blending.

# Slide credits

Most of these slides were adapted from:

- Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

- Fredo Durand (MIT).
- James Hays (Georgia Tech).

Some motivating examples

Gangster, Frankie Yale, killed by a drive-by in  
Brooklyn in 1928.





A tragic photo from 1959 after three-year-old Martha Cartagena was killed while riding her tricycle in Brooklyn

In 1958 there was a fatal fire at the Elkins Paper & Twine Co. on Wooster Street in SoHo. The building burned to the ground.





(c) Sergey Larenkov

Berlin 65 years later



*(c)Sergey Larenkov*



Berlin, 1945/2010, Mehringdamm

Forrest Gump (1994)



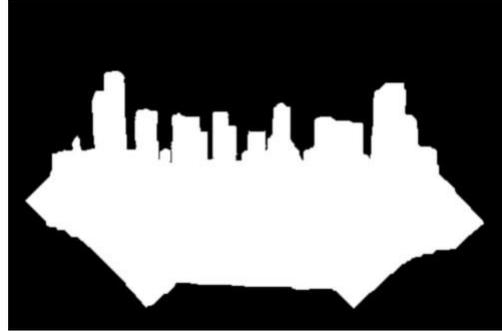
# Techniques for compositing

- Cut-and-paste.
- Alpha (linear) blending.
- Multi-band blending.
- Poisson blending.
- Seam stitching (next lecture).

Cut-and-paste

# Cut and paste procedure

1. Extract Sprites (e.g., using *Intelligent Scissors* in Photoshop)



2. Blend them into the composite (in the right order)



You may have also heard it as collaging



# Cut and paste



Other times, not so much.

What is wrong with this composite?

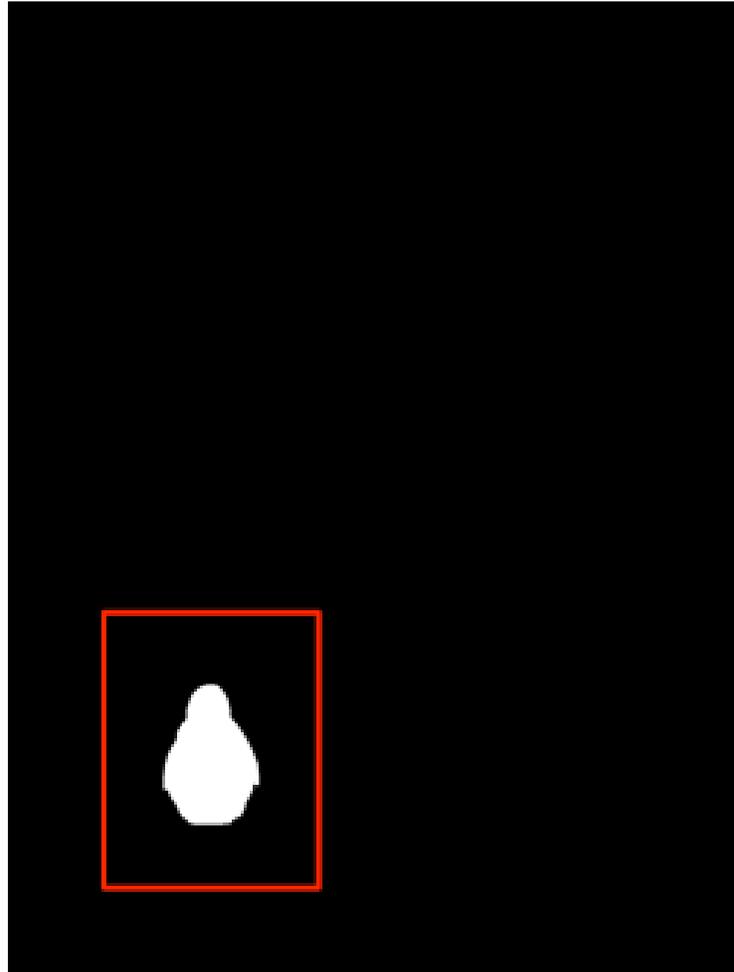
Alpha (linear) blending

# Alpha blending

foreground



mask



output



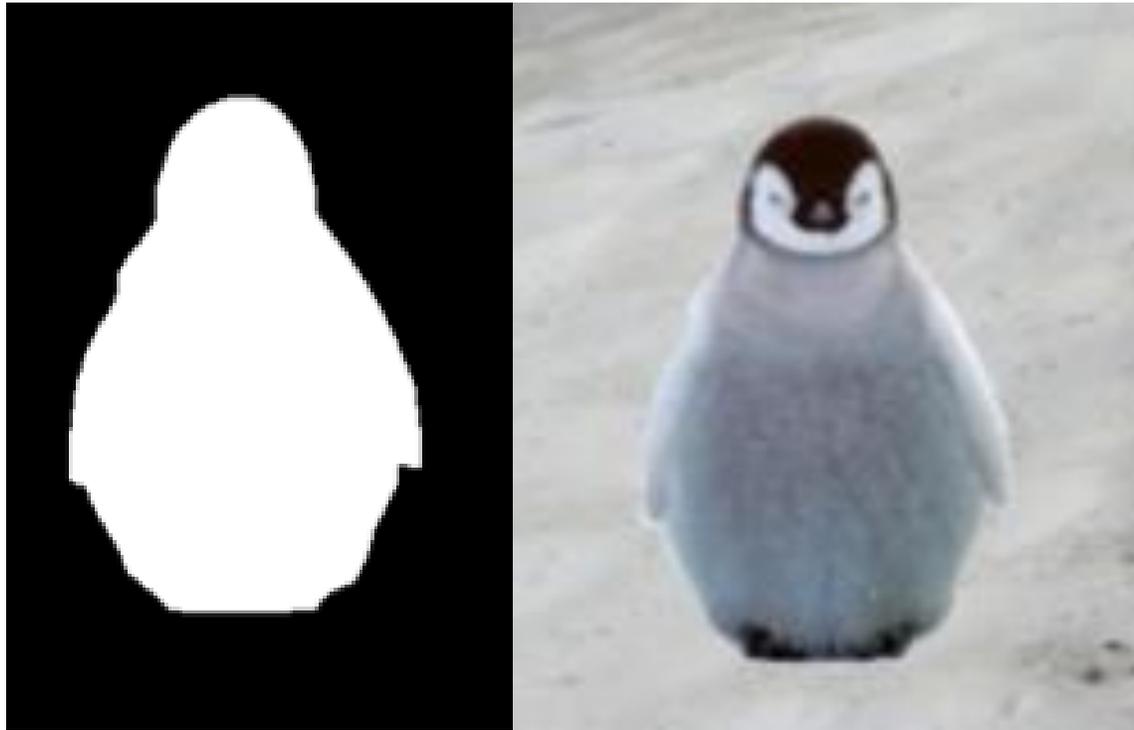
background



a.k.a. alpha matte  
or alpha composite

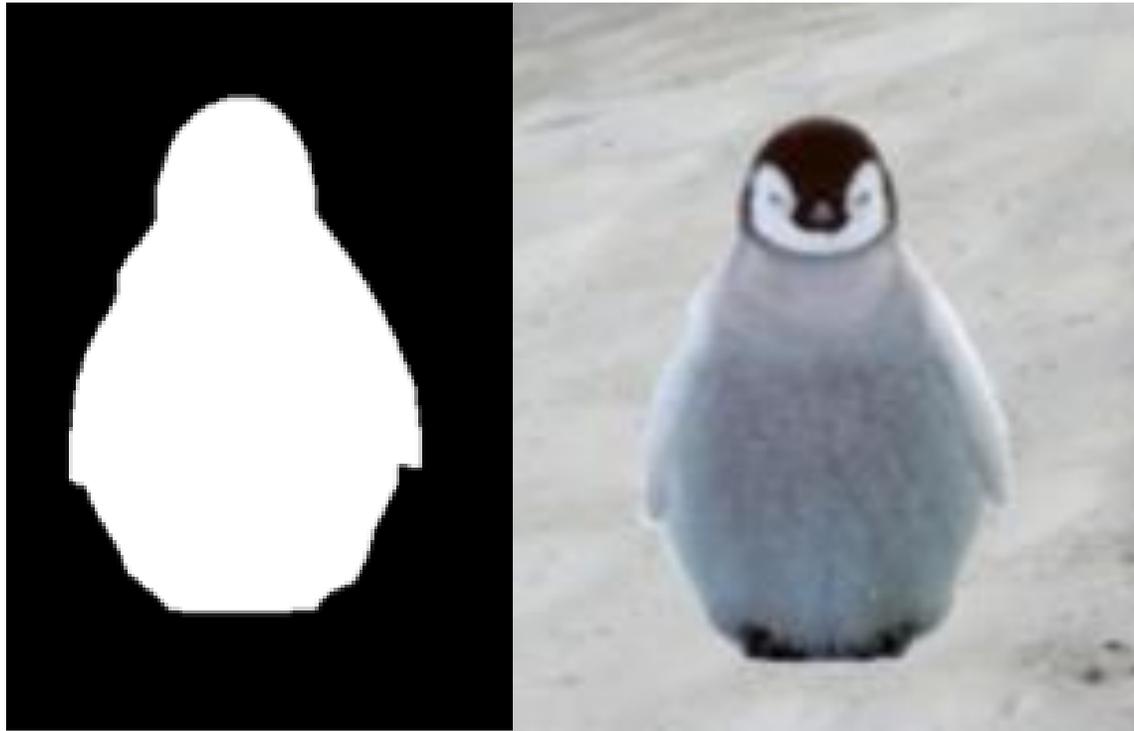
$$\text{output} = \text{foreground} * \text{mask} + \text{background} * (1 - \text{mask})$$

# Binary alpha mask



Does this look unnatural?

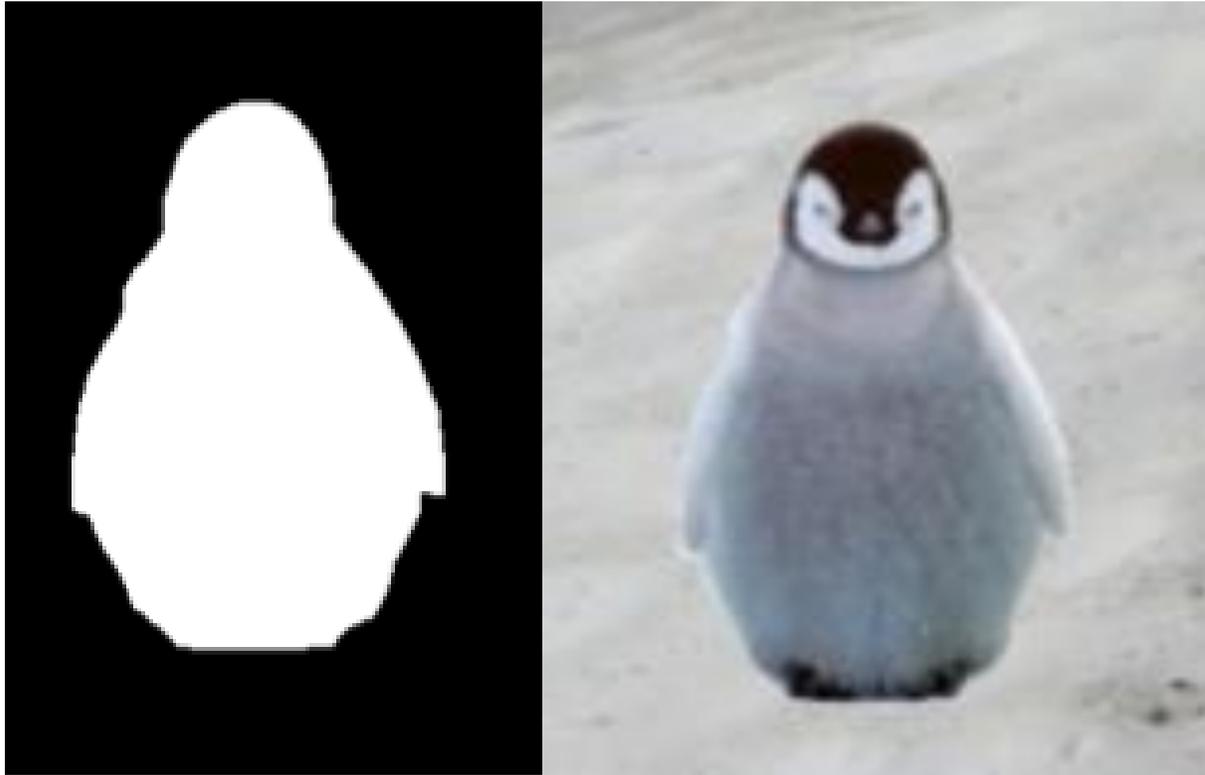
# Binary alpha mask



Does this look unnatural?  
How can we fix it?

# Non-binary alpha mask

binary alpha mask

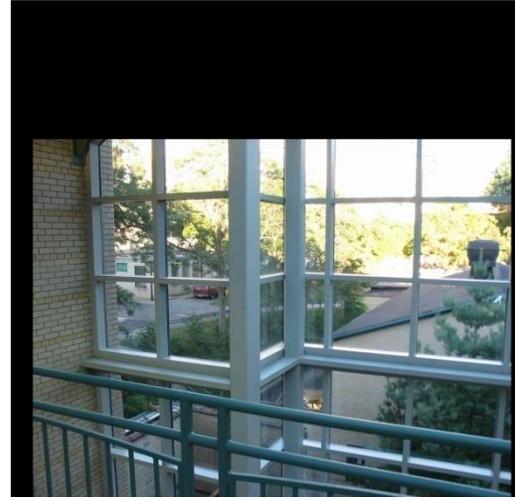


feathering (smoothed alpha)



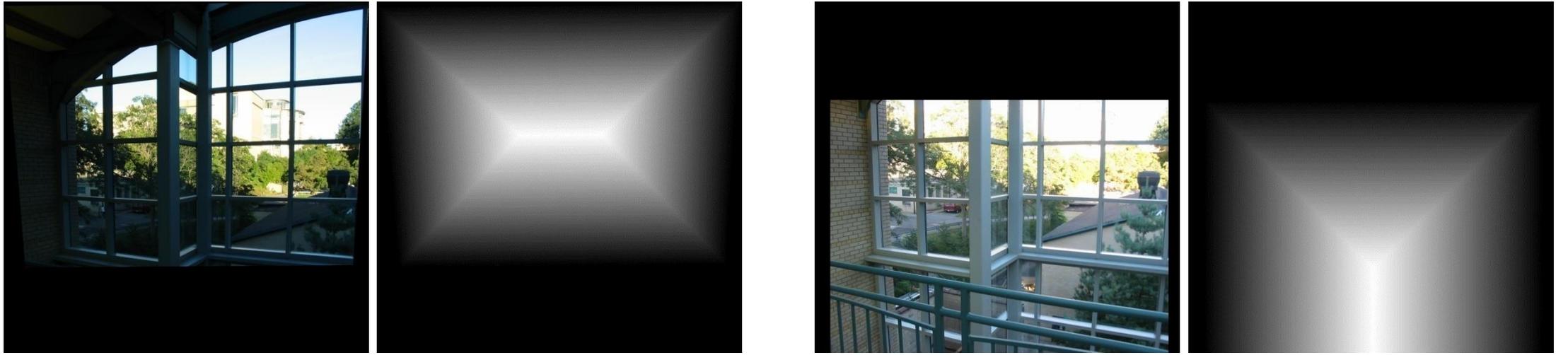
How would you implement feathering?

# Setting the alpha mask: center seam



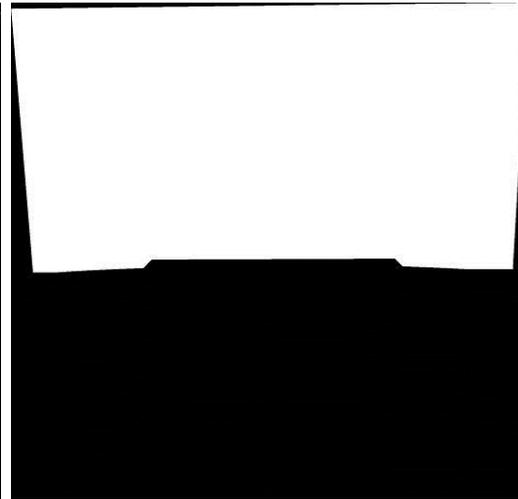
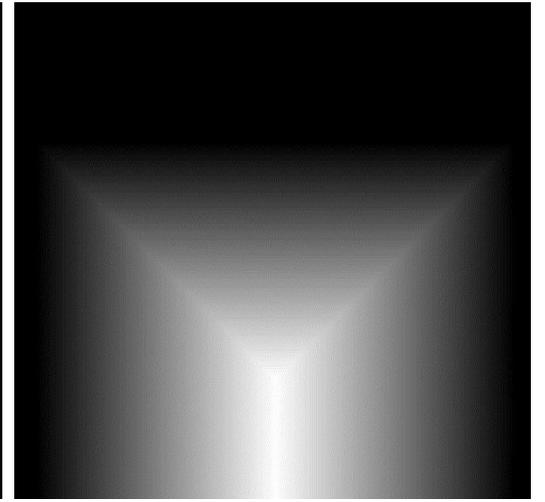
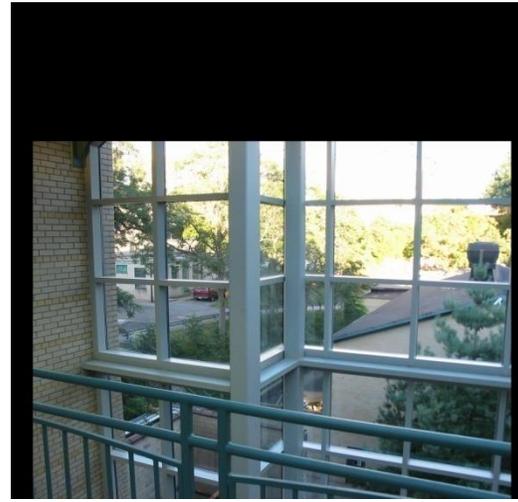
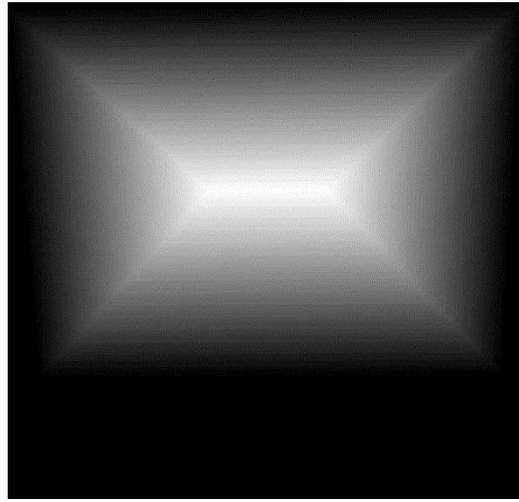
How would you create a binary alpha mask for these two images?

# Setting the alpha mask: center seam



Step 1: Compute their distance transform (`bwdist`)

# Setting the alpha mask: center seam



Step 2: set mask

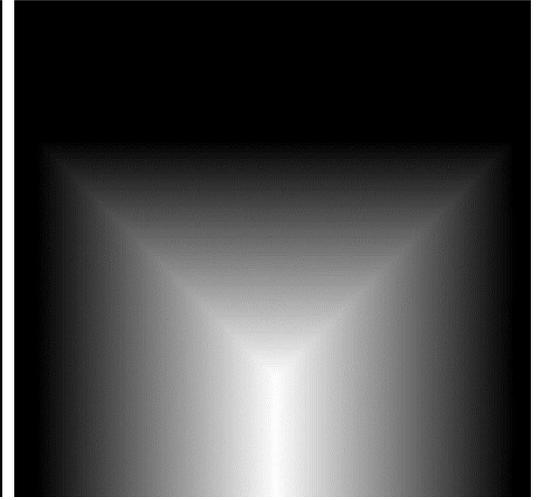
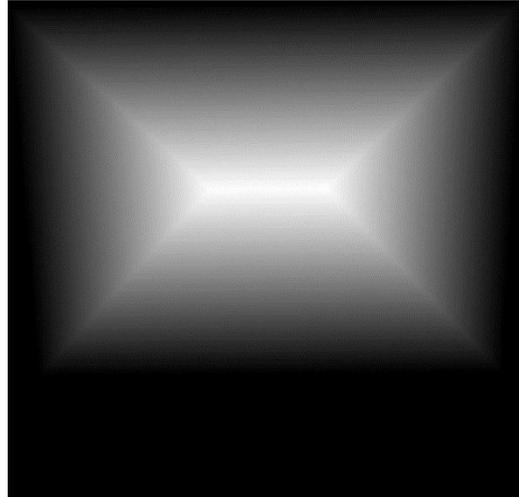
```
alpha = logical(dtrans1>dtrans2)
```

# Setting the alpha mask: center seam



Anything wrong with  
this alpha matte?

# Setting the alpha mask: center seam



Step 3: blur the mask

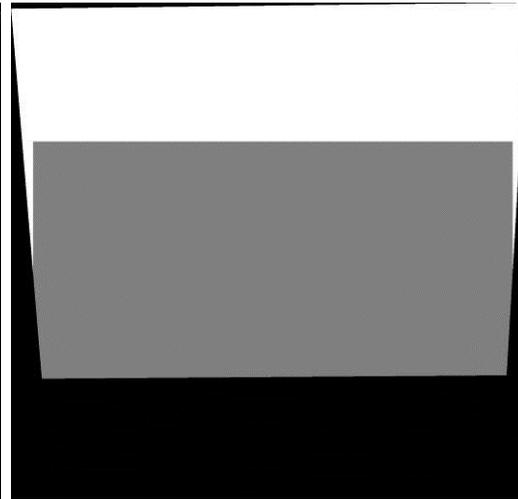
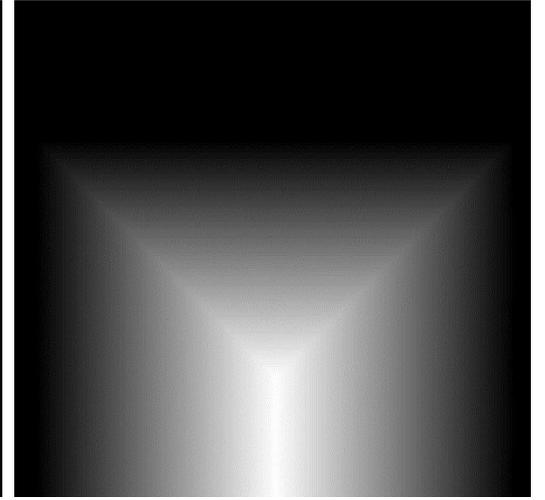
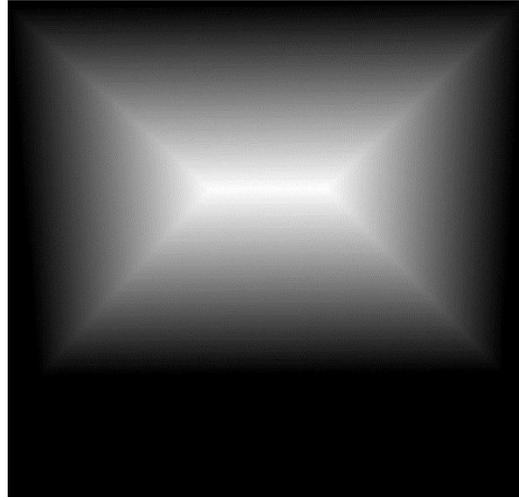
`alpha = blur(alpha)`

# Setting the alpha mask: center seam



Still doesn't look  
terribly good

# Setting the alpha mask: center seam



Step 4: go beyond blurring for non-binary

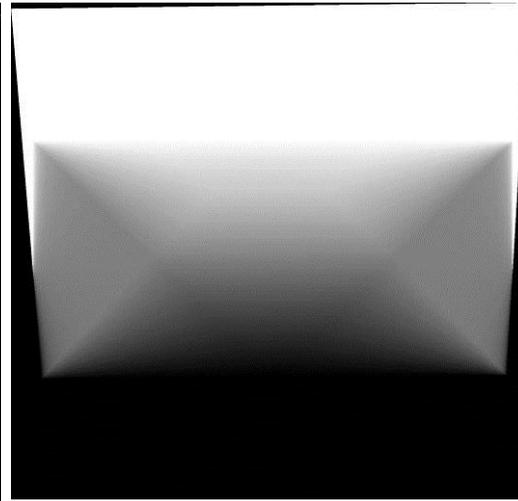
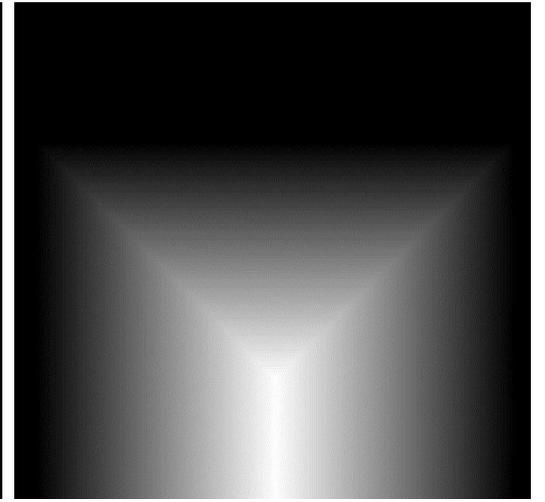
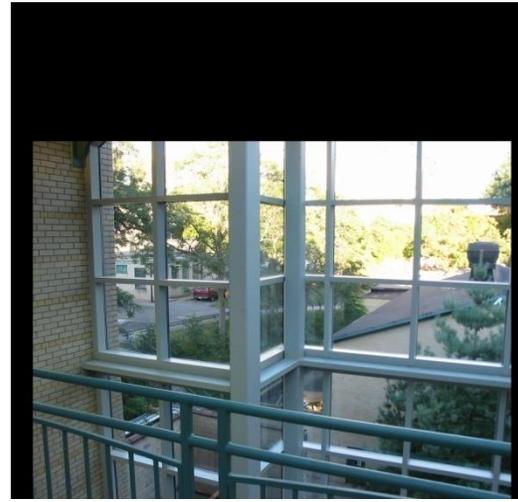
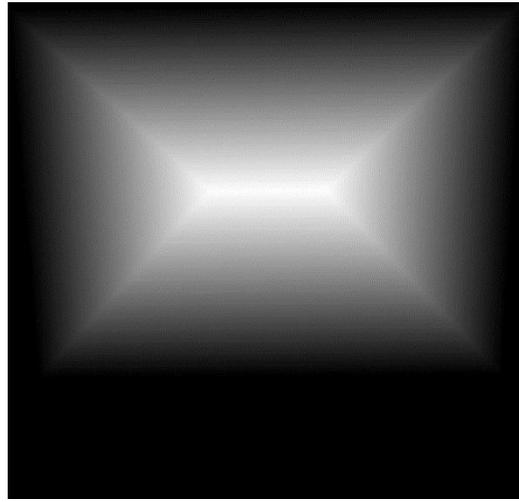
$\alpha = 0.5$  in overlap region

# Setting the alpha mask: center seam



Still not OK

# Setting the alpha mask: center seam



Step 5: more elaborate  
non-binary

$$\text{alpha} = \text{dtrans1} / (\text{dtrans1} + \text{dtrans2})$$

# Setting the alpha mask: center seam



Looks better but some dangers remain.

# Another blending example

Let's blend these two images...



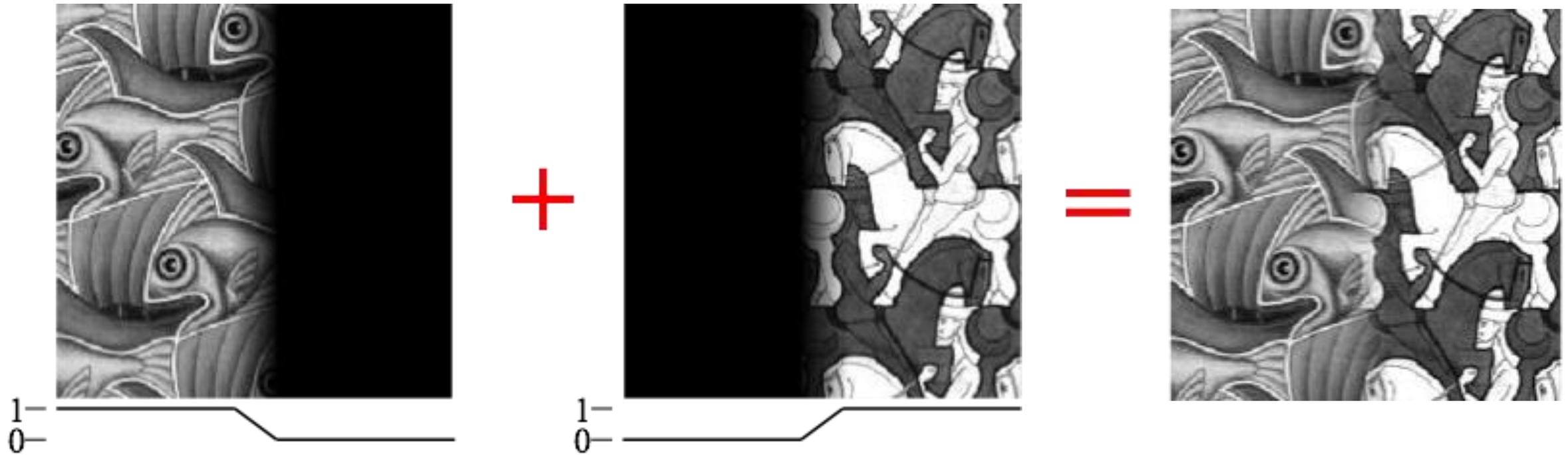
left side



right side

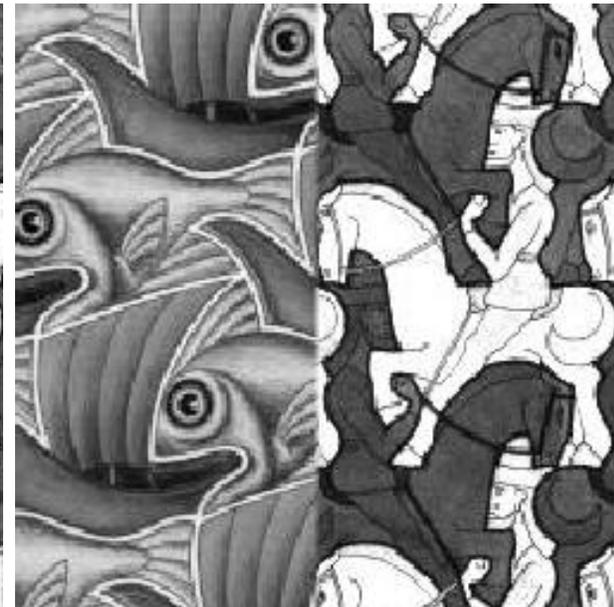
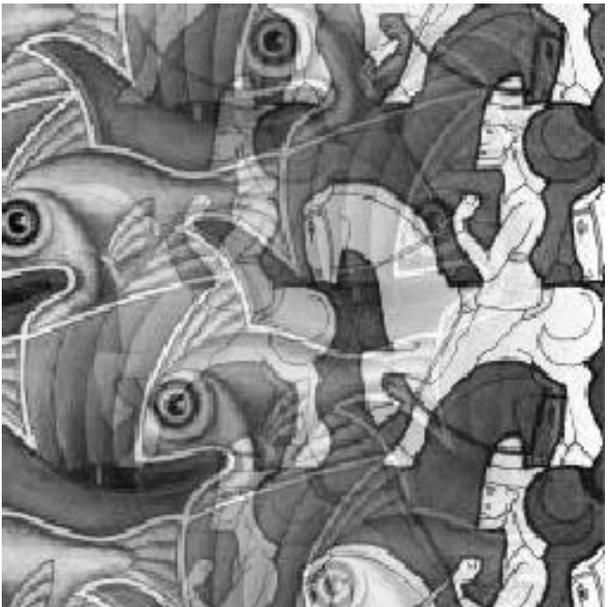
What kind of mask would you use?

# Another blending example



How would you select this window?

# Effects of different windows



Bad windows: ghosting.

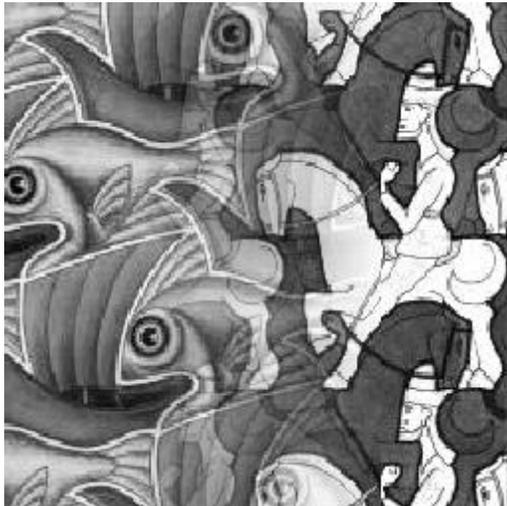
Good window: smooth but no ghosting.

Bad window: non-smooth seam.

# What is a good window size?



**To avoid discontinuities:**  
window = size of largest  
prominent feature



**To avoid ghosting:**  
window  $\leq 2 \times$  size of  
smallest prominent feature

# What is a good window size?

Fourier domain interpretation:

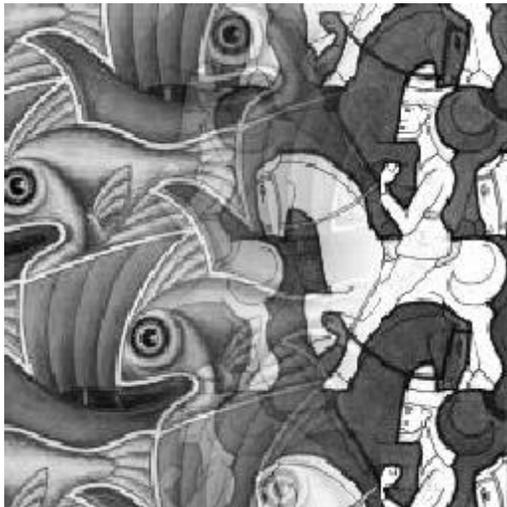
linear blending should work when:  
image frequency content occupies  
roughly one “octave” (power of two)

**To avoid discontinuities:**  
window = size of largest  
prominent feature

linear blending should work when:  
largest frequency  $\leq 2 \times$  size of smallest  
frequency

**To avoid ghosting:**  
window  $\leq 2 \times$  size of  
smallest prominent feature

What if the frequency spread is too wide?



# What is a good window size?

Fourier domain interpretation:

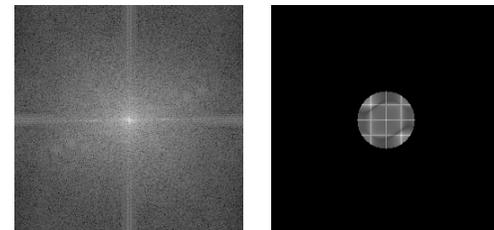
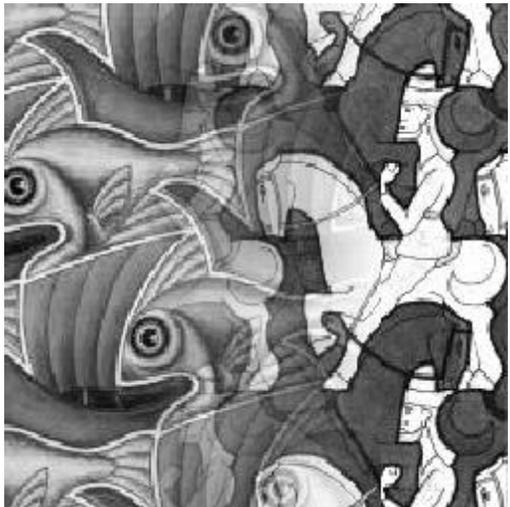
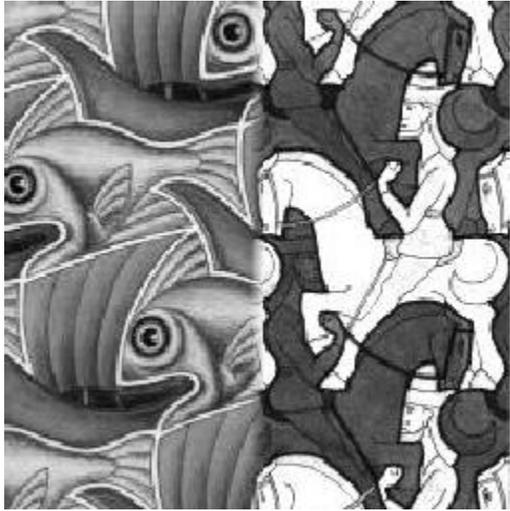
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window = size of largest  
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**To avoid ghosting:**  
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smallest prominent feature

linear blending should work when:  
largest frequency  $\leq 2 \times$  size of smallest  
frequency

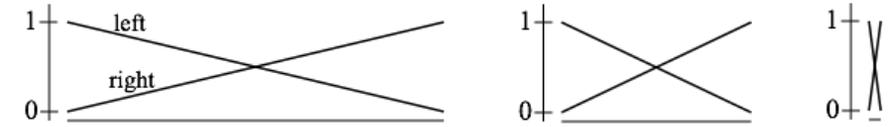
Most natural images have a very wide  
frequency spread. What do we do then?



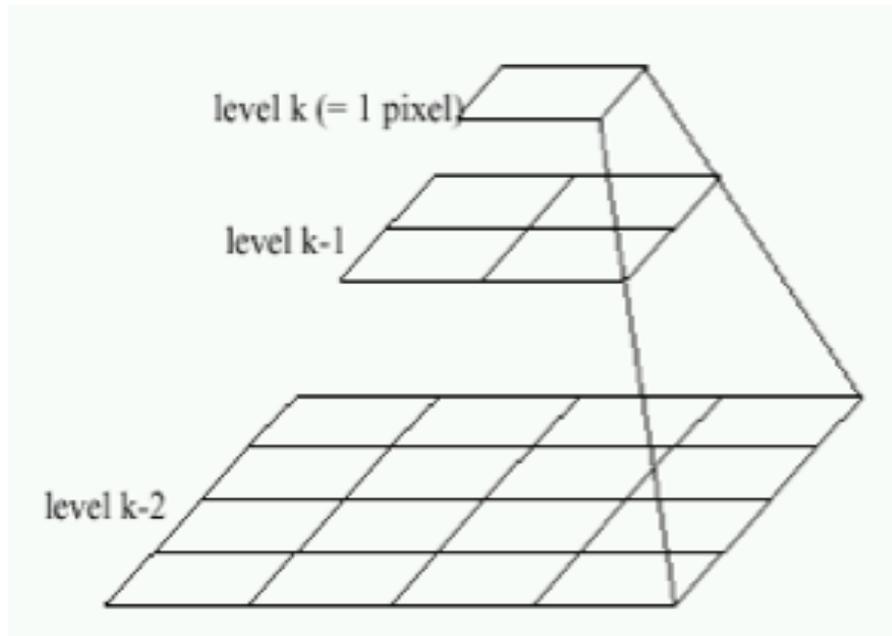
Multi-band blending

# Time to use pyramids again

At low frequencies, blend slowly to avoid seams  
At high frequencies, blend quickly to avoid ghosts



Which mask goes where?



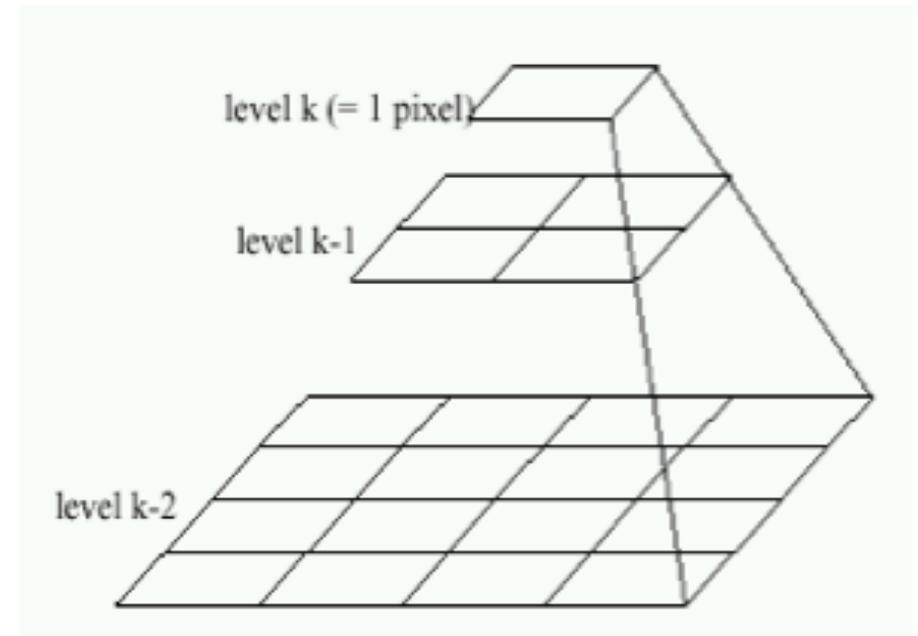
left image

?

?

?

alpha mask

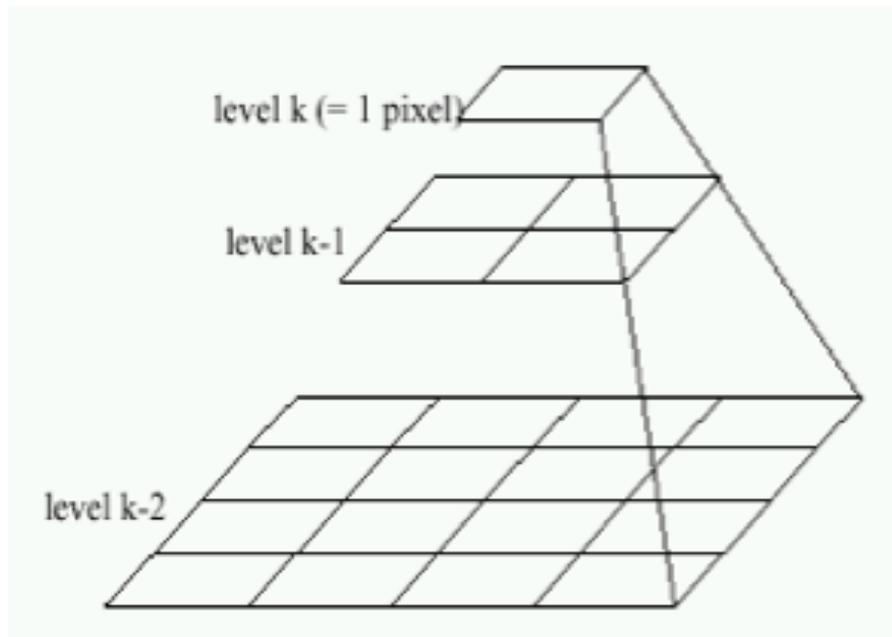


right image

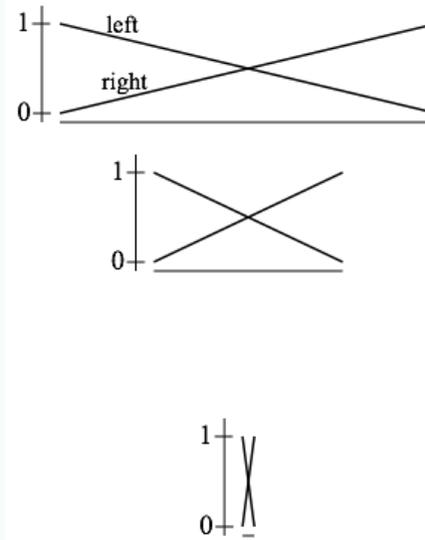
# Time to use pyramids again

At low frequencies, blend slowly to avoid seams

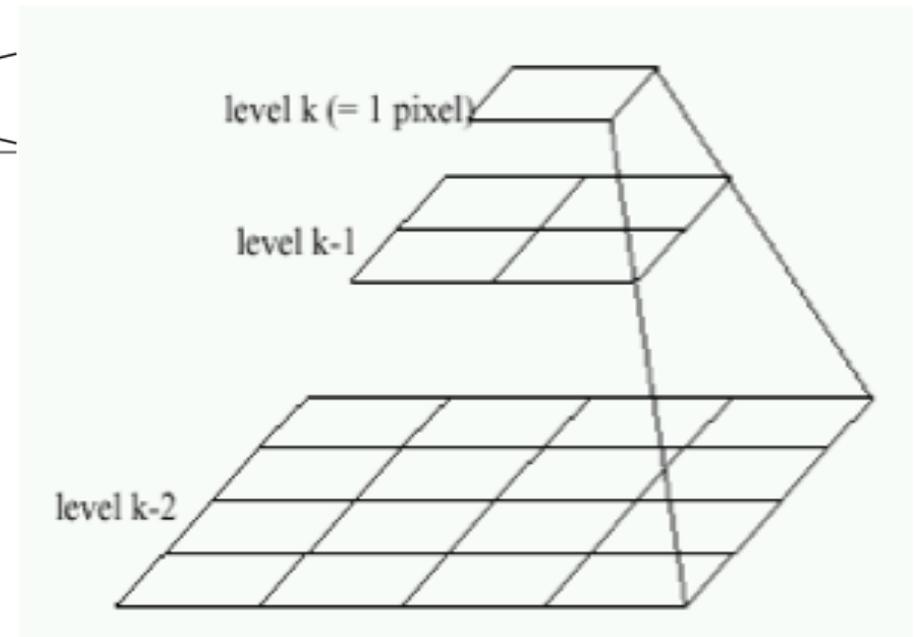
At high frequencies, blend quickly to avoid ghosts



left image

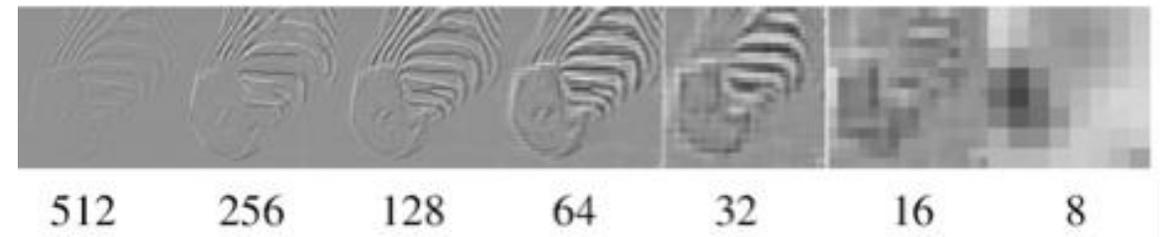
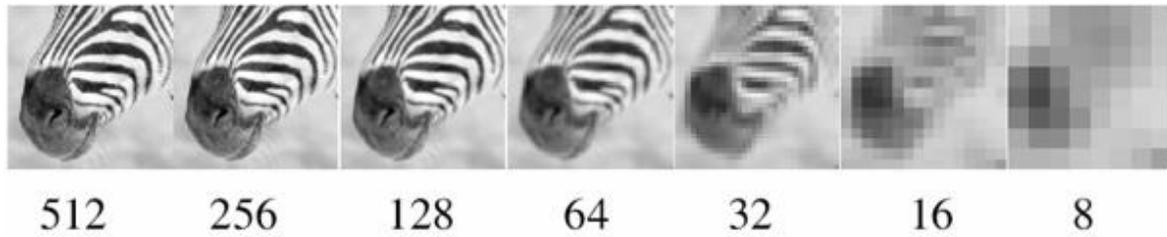


alpha mask



right image

# Remember our two types of pyramids



Gaussian pyramid



Laplacian pyramid

# Remember our two types of pyramids

1. Build Laplacian pyramids for each image

2. Blend each level of pyramid using region mask

$$L_{12}^i = L_1^i \cdot R^i + L_2^i \cdot (1 - R^i)$$

image 1  
at level i

image 2  
at level i

region mask  
at level i

How large should the blending region be at each level?

3. Collapse the pyramid to get the final blended image

# Remember our two types of pyramids

1. Build Laplacian pyramids for each image

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$$L_{12}^i = L_1^i \cdot R^i + L_2^i \cdot (1 - R^i)$$

image 1  
at level i

image 2  
at level i

region mask  
at level i

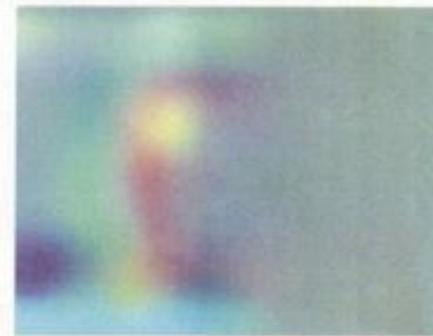
How large should the blending  
region be at each level?

About the size of that level's blur

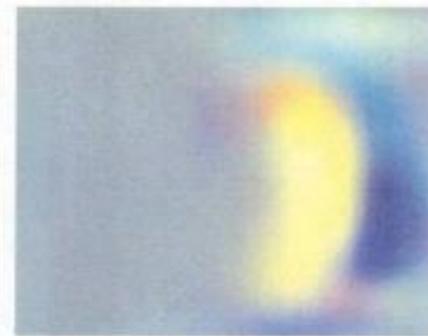
3. Collapse the pyramid to get the final blended image

# Multi-band blending using the Laplacian pyramid

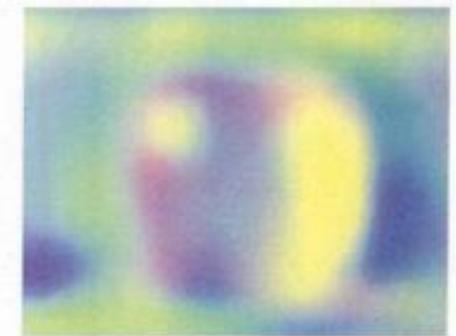
Laplacian level 4



(e)

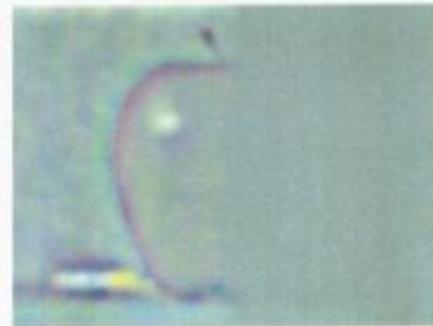


(g)

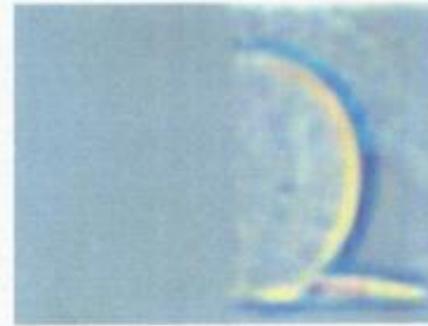


(k)

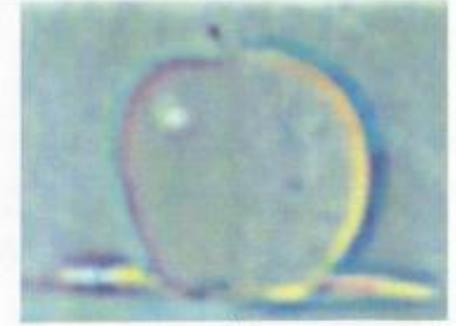
Laplacian level 2



(b)

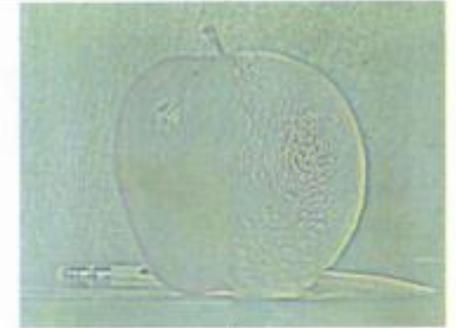
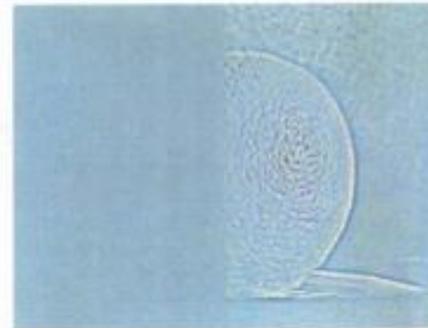
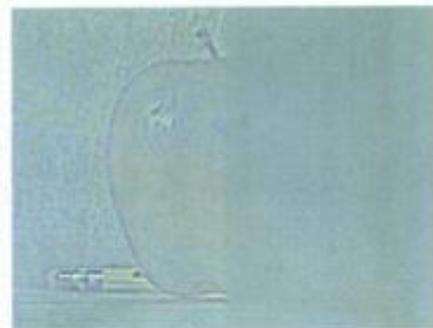


(f)



(j)

Laplacian level 0

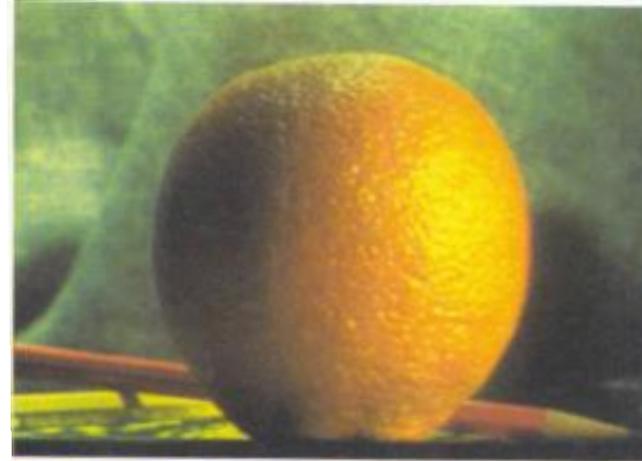
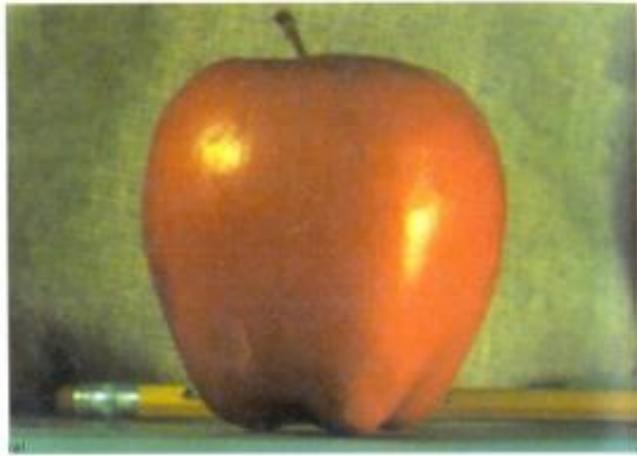


left pyramid

right pyramid

blended pyramid

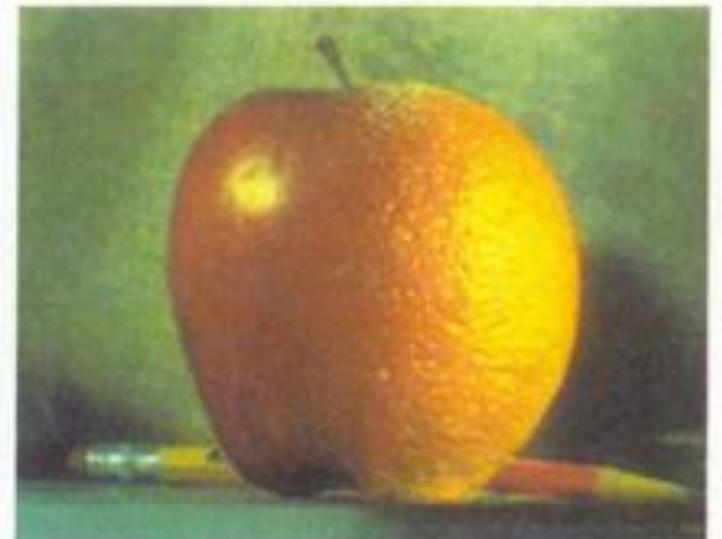
# A famous result (for its time)



(d)

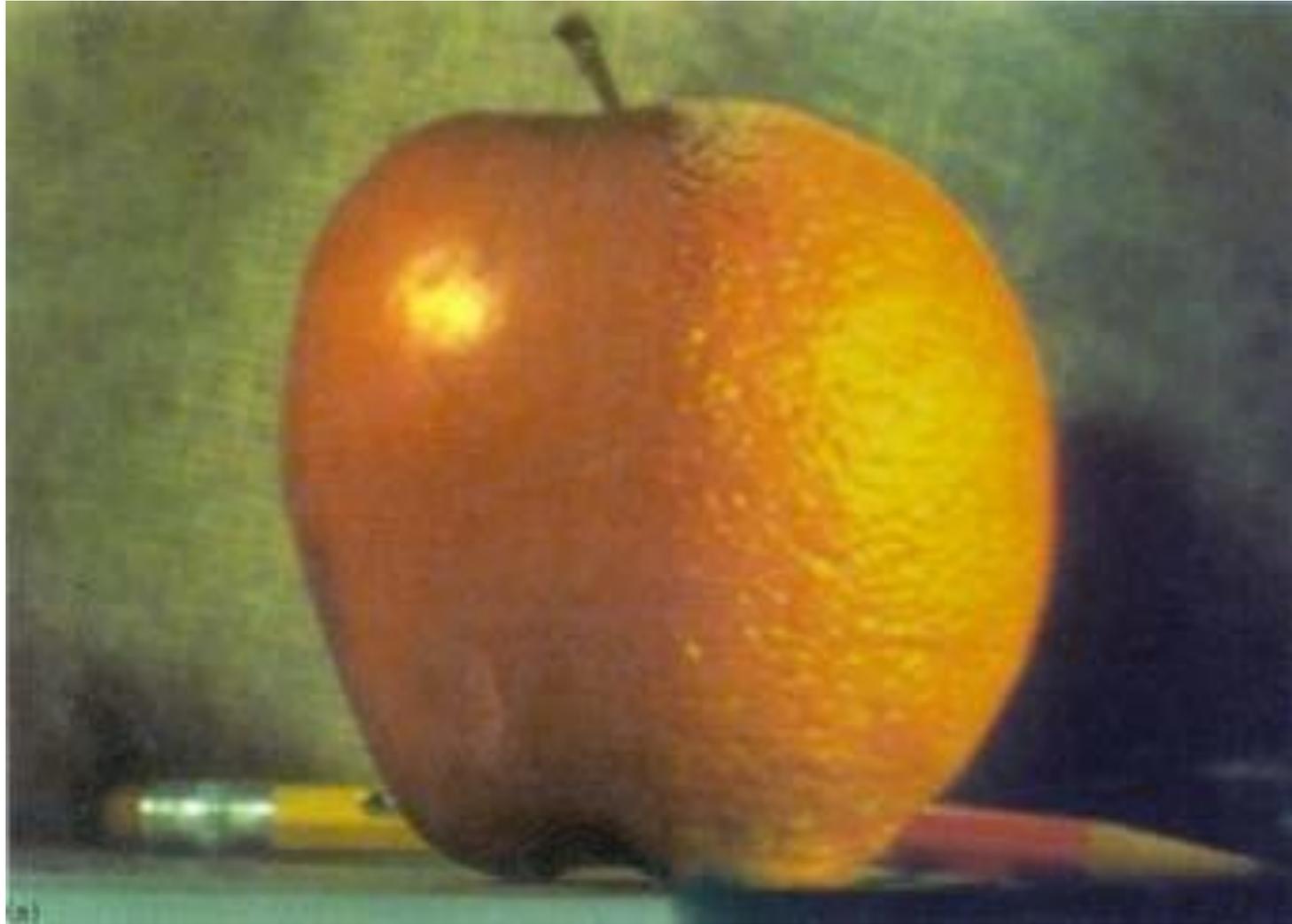


(h)

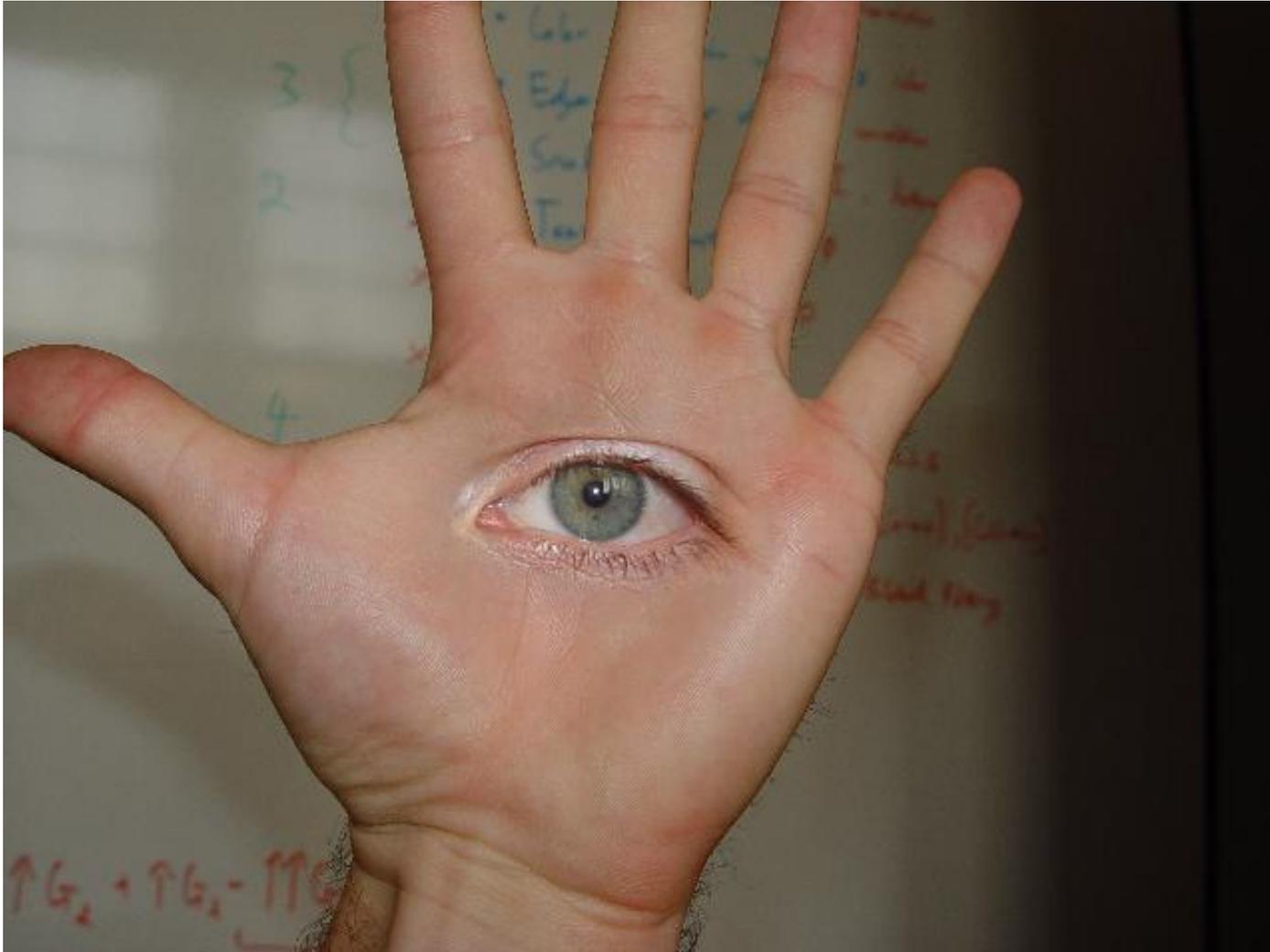


(l)

A famous result (for its time)



# A creepier result

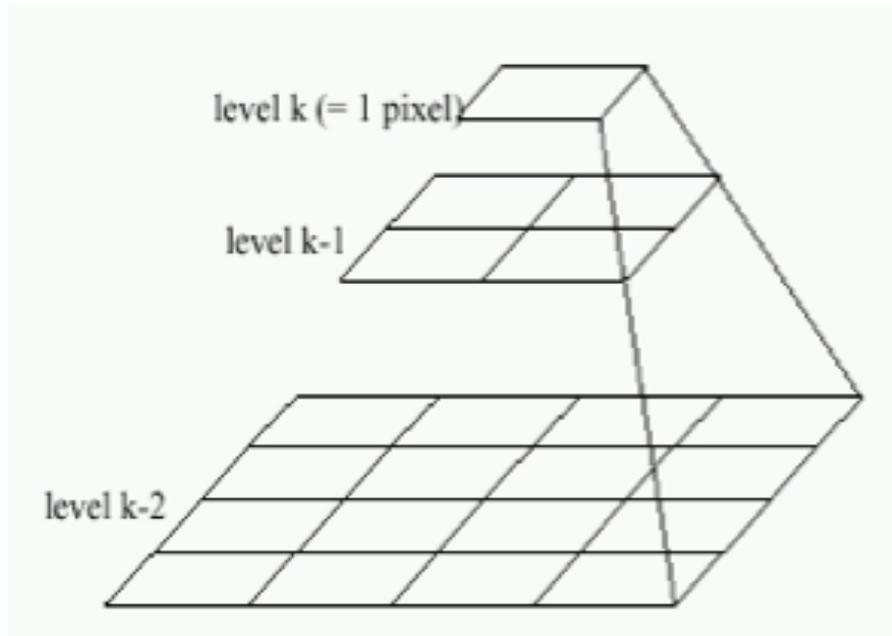


Can we get the same result with less computation?

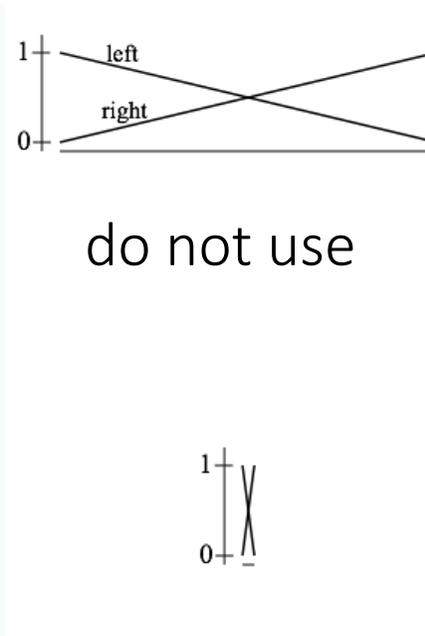
# Two-band blending

Only use two bands: high frequency and low frequency

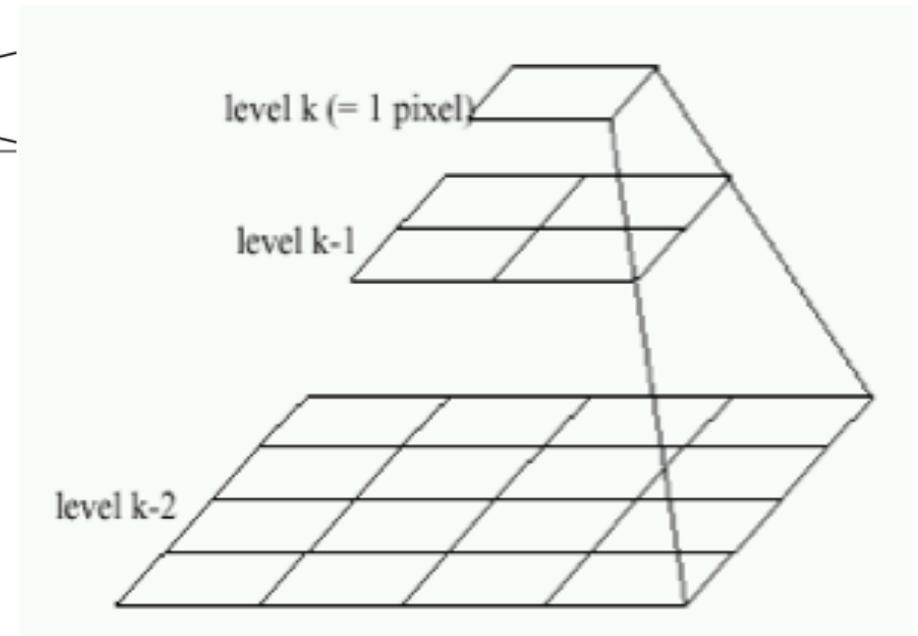
- Blend low frequency with smooth alpha
- Blend high frequency with binary alpha



left image



alpha mask



right image

# Example: blending panoramas

original  
collage



blended  
collage



# Example: blending panoramas

low  
frequency  
blend



high  
frequency  
blend



# Linear blending



# Two-band blending



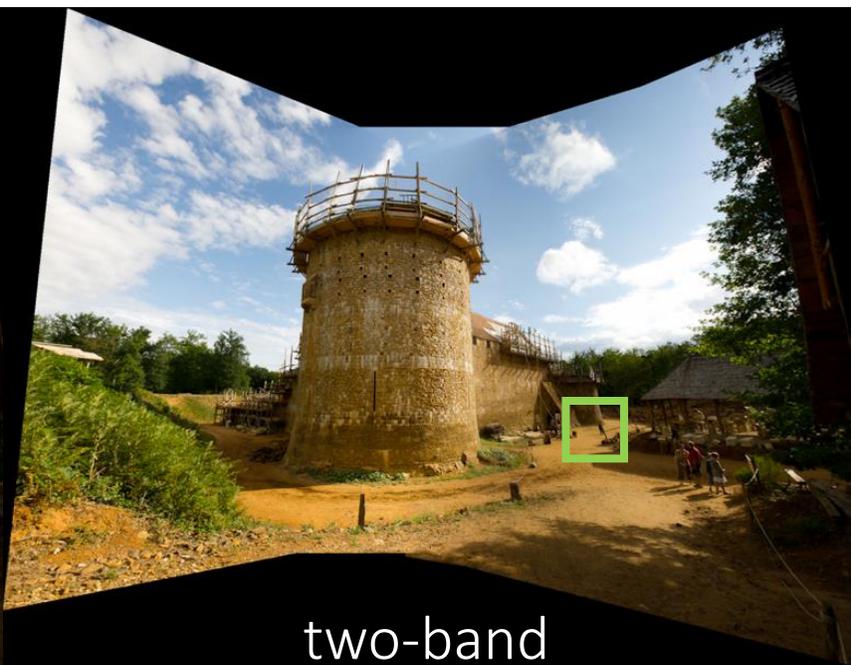
# One more comparison



copy-paste



linear



two-band



# Why do these images look weirdly cropped?



They were warped using homographies before being aligned.

Homework 6: autostitching

Poisson blending

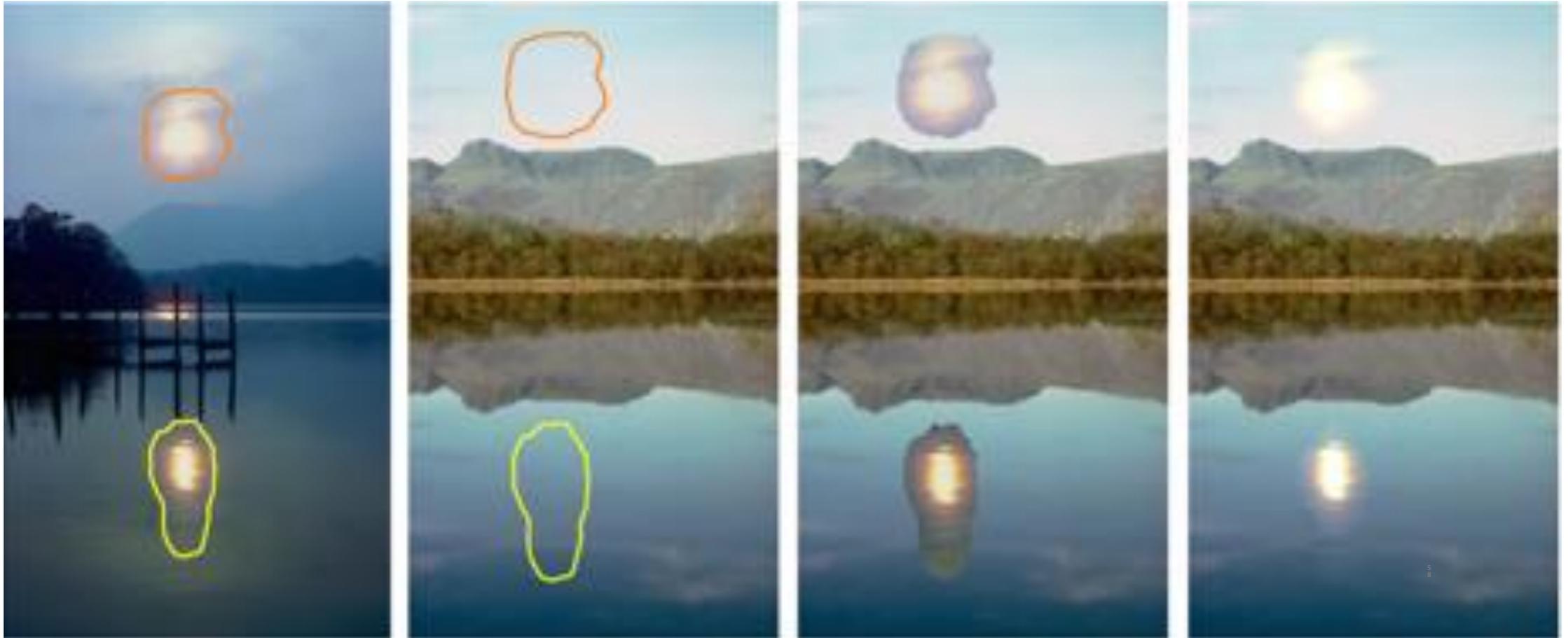
# Someone leaked season 8 of Game of Thrones



or, more likely, they made some creative use of Poisson blending

# Key idea

When blending, retain the gradient information as best as possible



source

destination

copy-paste

Poisson blending

# Example

How come the colors get smoothed out after blending?



originals

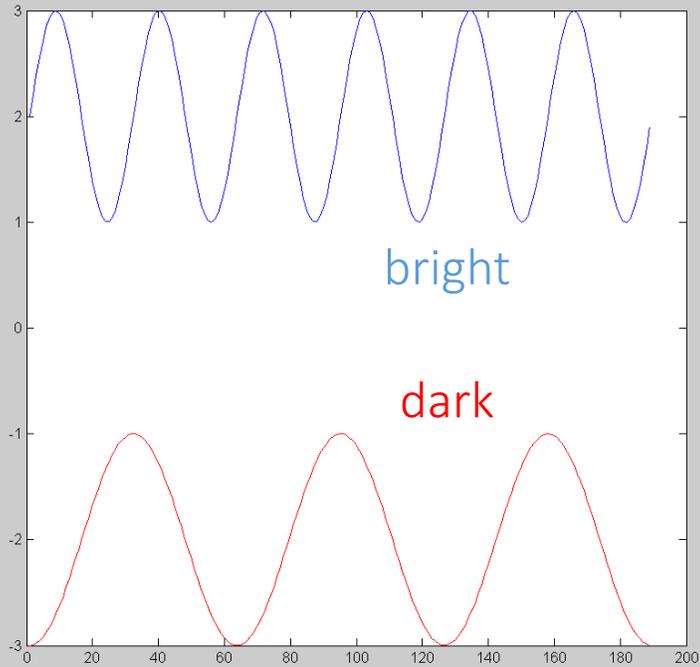


copy-paste

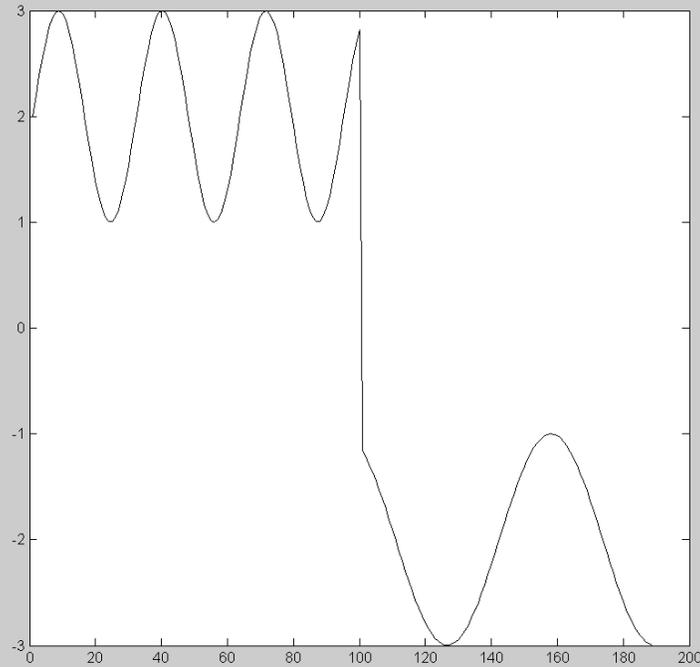


Poisson blending

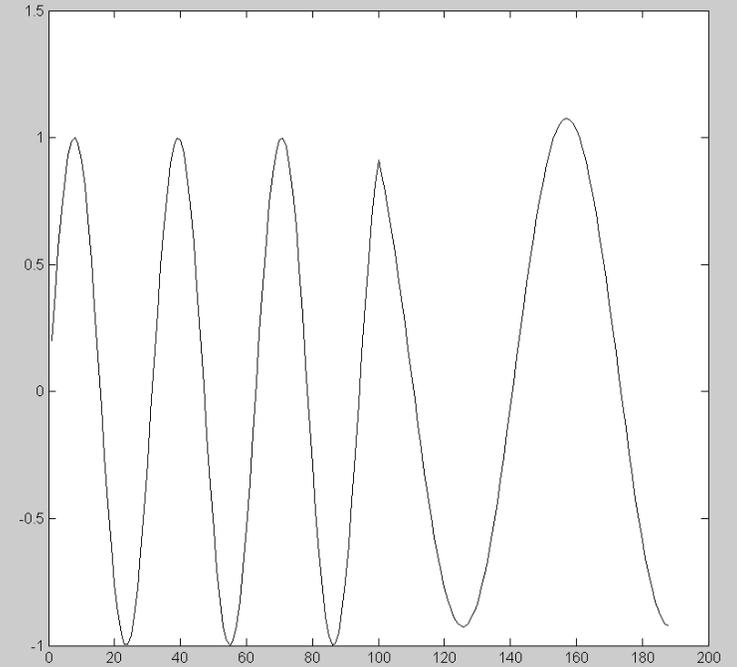
# Poisson blending: 1D example



two signals



regular blending



blending derivatives

# Warning: math ahead

Also note: you'll implement this for homework 3.

# Definitions and notation



## Notation

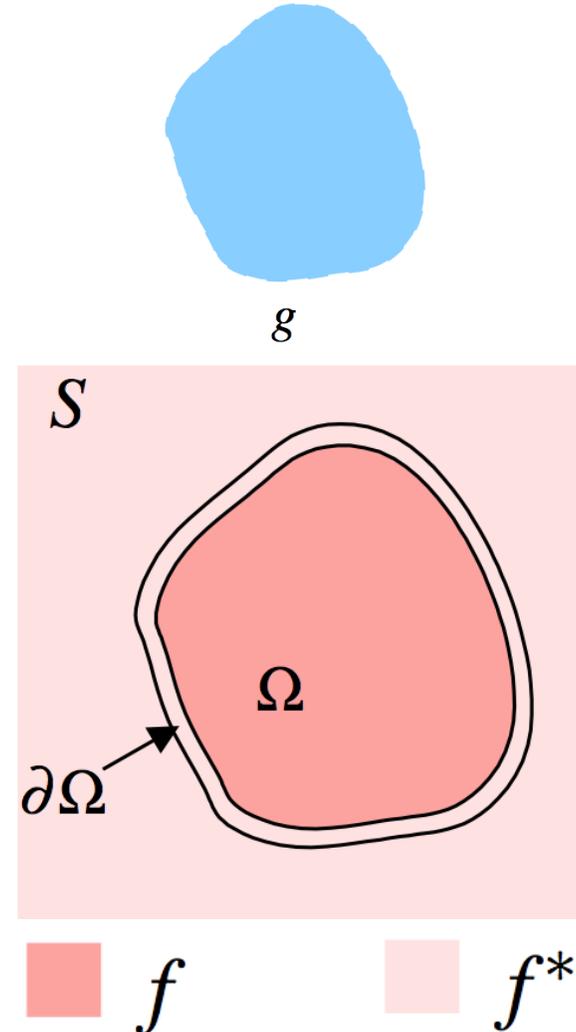
$g$ : source function

$S$ : destination

$\Omega$ : destination domain

$f$ : interpolant function

$f^*$ : destination function



Which one is the unknown?

# Definitions and notation



## Notation

$g$ : source function

$S$ : destination

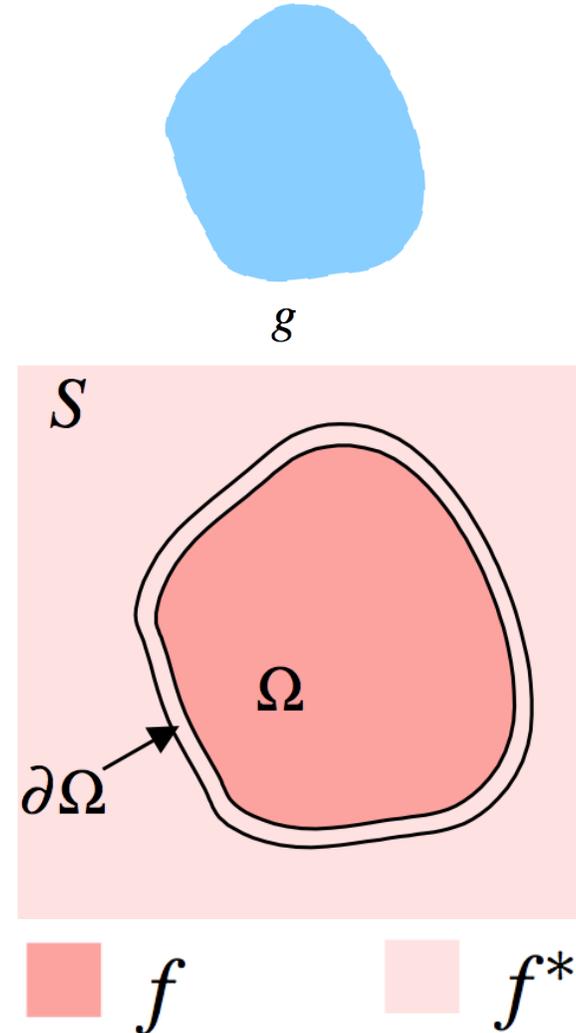
$\Omega$ : destination domain

$f$ : interpolant function

$f^*$ : destination function

How should we determine  $f$ ?

- should it look like  $g$ ?
- should it look like  $f^*$ ?



# Interpolation criterion

“Variational” means optimization where the unknown is an entire function

Variational problem

$$\min_f \iint_{\Omega} |\nabla f - \mathbf{v}|^2 \quad \text{with} \quad f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

what does this term do?

what does this term do?

Recall ...

Image gradient

$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

is this known?

$$\mathbf{v} = (u, v) = \nabla g$$

# Interpolation criterion

“Variational” means optimization where the unknown is an entire function

Variational problem

$$\min_f \iint_{\Omega} |\nabla f - \mathbf{v}|^2 \quad \text{with} \quad f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

gradient of  $f$  looks like gradient of  $g$

$f$  is equivalent to  $f^*$  at the boundaries

Recall ...

**Image gradient**

$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

Yes, since the source function  $g$  is known

$$\mathbf{v} = (u, v) = \nabla g$$

# Equivalently

This is where *Poisson*  
blending comes from

Poisson equation (with Dirichlet boundary conditions)

$$\Delta f = \operatorname{div} \mathbf{v} \quad \text{over } \Omega, \quad \text{with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

what does this term do?

**Gradient**  $\mathbf{v} = (u, v) = \nabla g$

**Laplacian**  $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

**Divergence**  $\operatorname{div} \mathbf{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$$\begin{aligned} \operatorname{div} \mathbf{v} &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \\ &= \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \\ &= \Delta g \end{aligned}$$

# Equivalently

This is where *Poisson*  
blending comes from

Poisson equation (with Dirichlet boundary conditions)

$$\Delta f = \operatorname{div} \mathbf{v} \quad \text{over } \Omega, \quad \text{with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

Laplacian of  $f$  same as  $g$

**Gradient**  $\mathbf{v} = (u, v) = \nabla g$

**Laplacian**  $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

**Divergence**  $\operatorname{div} \mathbf{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

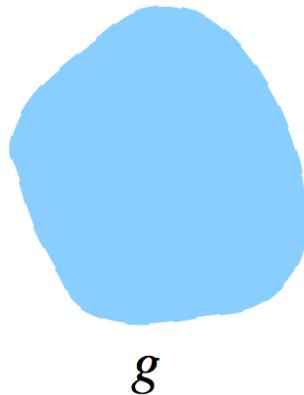
$$\begin{aligned} \operatorname{div} \mathbf{v} &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \\ &= \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \\ &= \Delta g \end{aligned}$$

# Equivalently

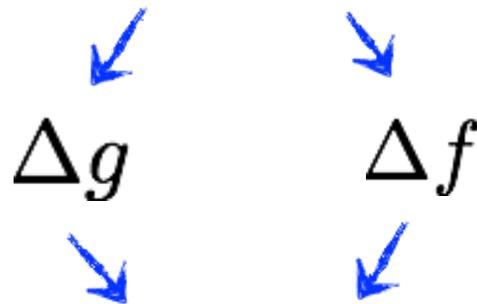
This is where *Poisson*  
blending comes from

Poisson equation (with Dirichlet boundary conditions)

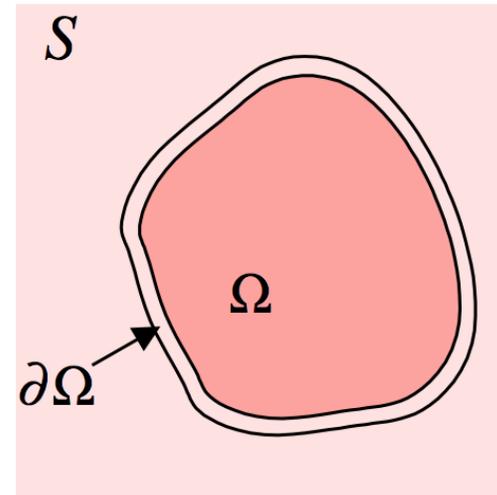
$$\Delta f = \text{div } \mathbf{v} \quad \text{over } \Omega, \quad \text{with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$



so make these guys ...



the same



How can we do this?

# Equivalently

This is where *Poisson* blending comes from

Poisson equation (with Dirichlet boundary conditions)

$$\Delta f = \operatorname{div} \mathbf{v} \quad \text{over } \Omega, \quad \text{with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

So for each pixel  $p$ , do:

$$\Delta f_p = \Delta g_p$$

Or for discrete images:

$$4f_p - \sum_{q \in N_p} f_q = 4g_p - \sum_{q \in N_p} g_q$$

How did we go from one to the other?

# Equivalently

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Recall...

Laplace filter

0	1	0
1	-4	1
0	1	0

What's known and what's unknown?

# Equivalently

This is where *Poisson* blending comes from

Poisson equation (with Dirichlet boundary conditions)

$$\Delta f = \operatorname{div} \mathbf{v} \quad \text{over} \quad \Omega, \quad \text{with} \quad f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

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Recall...

Laplace filter

0	1	0
1	-4	1
0	1	0

$f$  is unknown except at the boundary  
 $g$  and its Laplacian are known

# We can rewrite this as

linear equation  
of  $N$  variables

$$4f_p - \sum_{q \in N_p} f_q = 4g_p - \sum_{q \in N_p} g_q$$

one for each pixel  
in destination

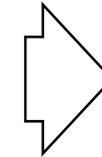
In vector form:

$$[0 \dots 1 \dots 1 \ 4 \ 1 \dots 1 \dots 0]$$

What is this? ↗

(each pixel adds another 'sparse' row here)

$$\begin{bmatrix} f_1 \\ \vdots \\ f_{q_1} \\ \vdots \\ f_{q_2} \\ f_p \\ f_{q_3} \\ \vdots \\ f_{q_4} \\ \vdots \\ f_N \end{bmatrix} = \begin{bmatrix} \Delta g_1 \\ \vdots \\ \Delta g_{q_1} \\ \vdots \\ \Delta g_{q_2} \\ \Delta g_p \\ \Delta g_{q_3} \\ \vdots \\ \Delta g_{q_4} \\ \vdots \\ \Delta g_N \end{bmatrix}$$



Linear system of equations

$$\mathbf{A}f = \mathbf{b}$$

How would you solve this?

WARNING: requires special treatment at the borders  
(target boundary values are same as source )

# Solving the linear system

Convert the system to a linear least-squares problem:

$$E_{\text{LLS}} = \|\mathbf{A}f - \mathbf{b}\|^2$$

Expand the error:

$$E_{\text{LLS}} = f^\top (\mathbf{A}^\top \mathbf{A}) f - 2f^\top (\mathbf{A}^\top \mathbf{b}) + \|\mathbf{b}\|^2$$

Minimize the error:

Set derivative to 0  $(\mathbf{A}^\top \mathbf{A})f = \mathbf{A}^\top \mathbf{b}$

Solve for x  $f = (\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top \mathbf{b}$

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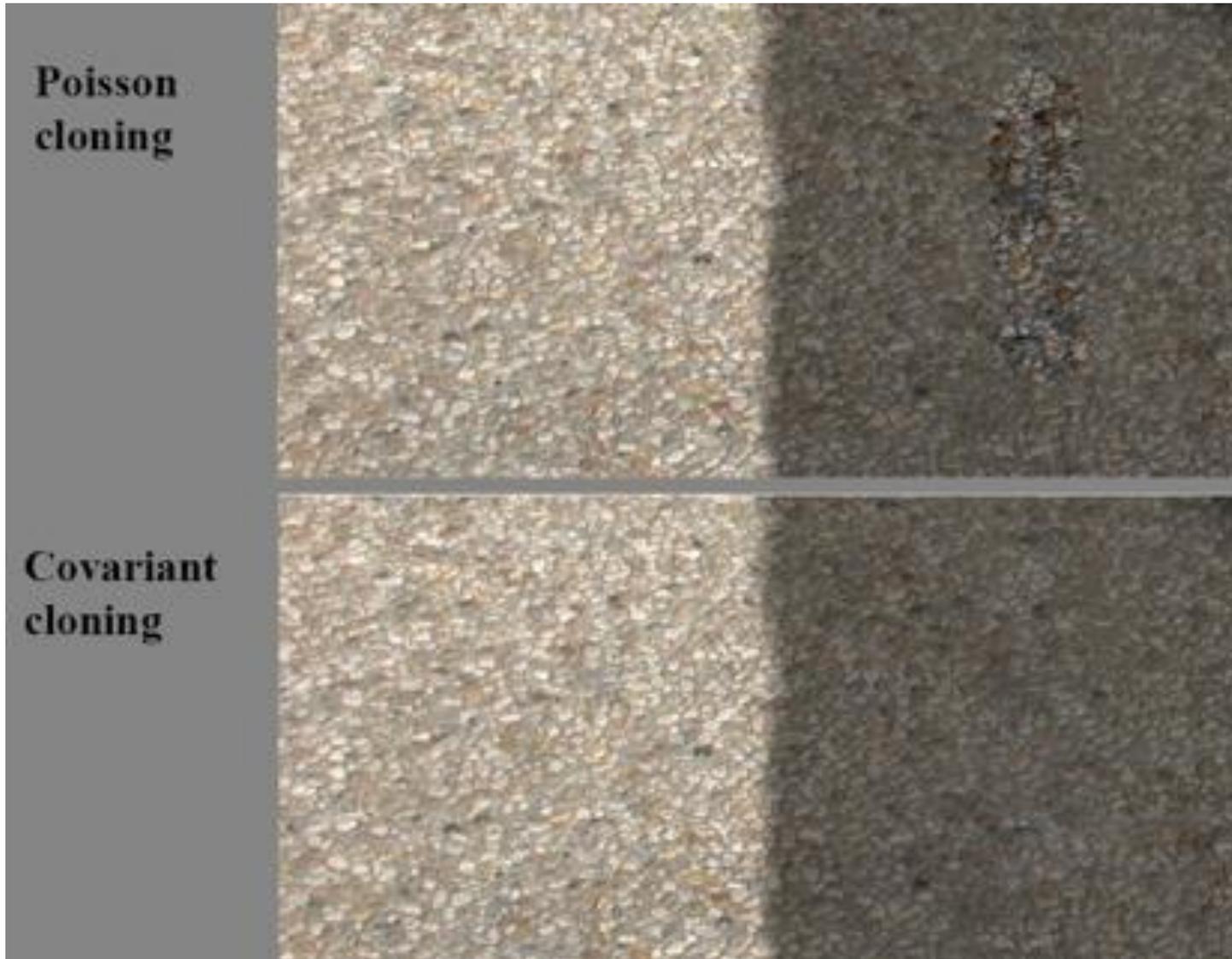
Solve for x  $f = (\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top \mathbf{b}$  ←

In Matlab:

$$f = A \setminus b$$

Note: You almost never want to compute the inverse of a matrix.

# Photoshop's "healing brush"



- Slightly more advanced version of what we covered here:
- Uses higher-order derivatives

# Contrast problem



Loss of contrast when pasting from dark to bright:

- Contrast is a multiplicative property.
- With Poisson blending we are matching linear differences.



# Contrast problem



Loss of contrast when pasting from dark to bright:

- Contrast is a multiplicative property.
- With Poisson blending we are matching linear differences.

Solution: Do blending in log-domain.



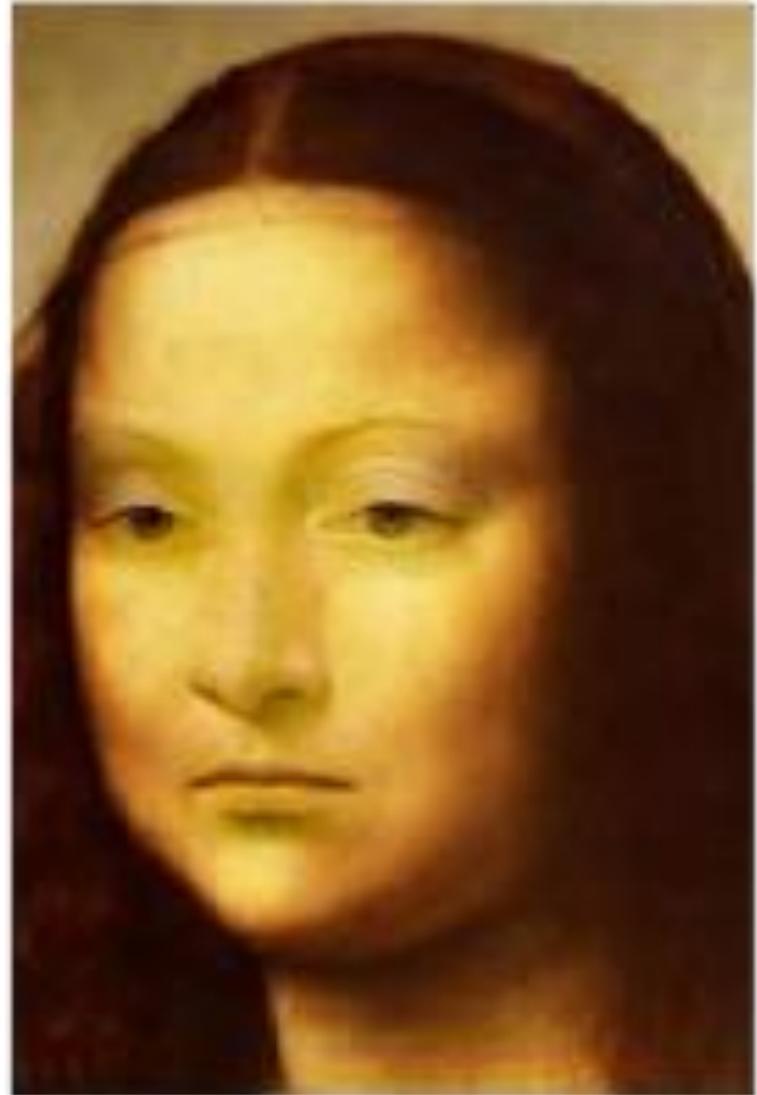
# More blending



originals



copy-paste

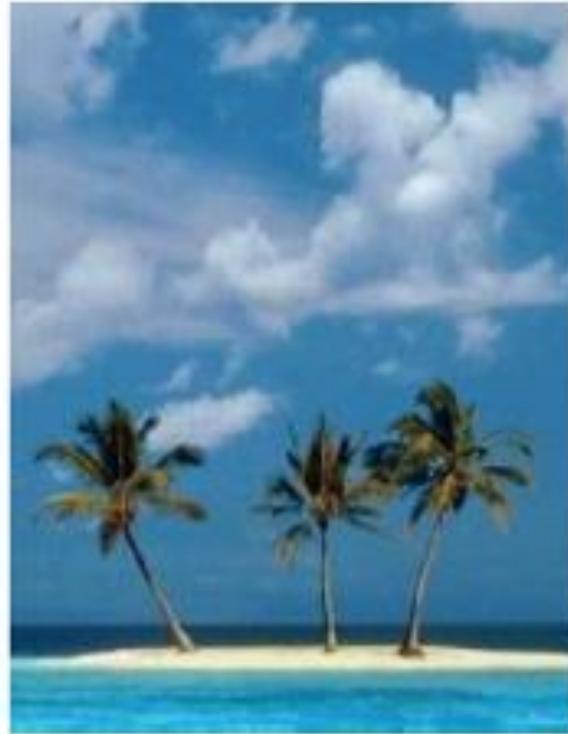


Poisson blending

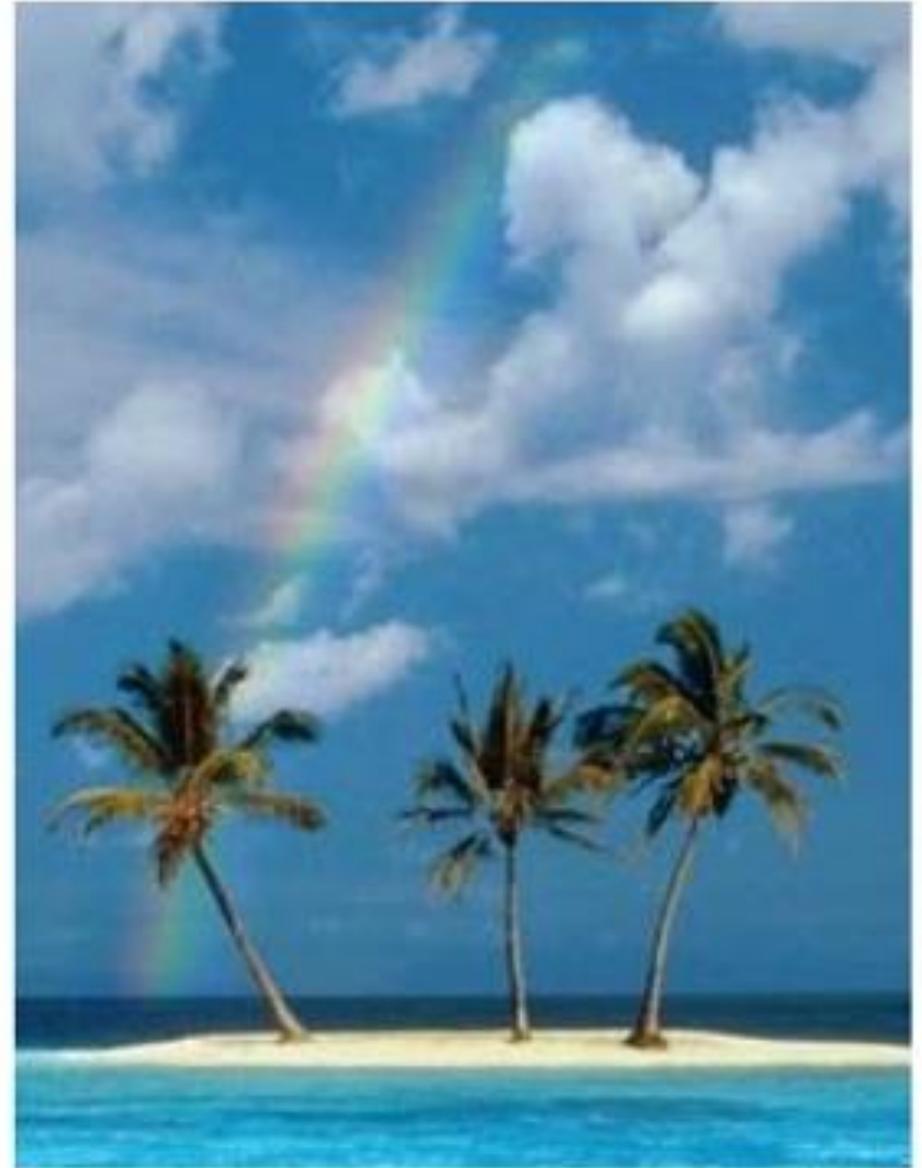
# Blending transparent objects



source



destination



# Blending objects with holes



(a) color-based cutout and paste



(b) seamless cloning



(c) seamless cloning and destination averaged



(d) mixed seamless cloning

# Editing



# Concealment



How would you do this with Poisson blending?



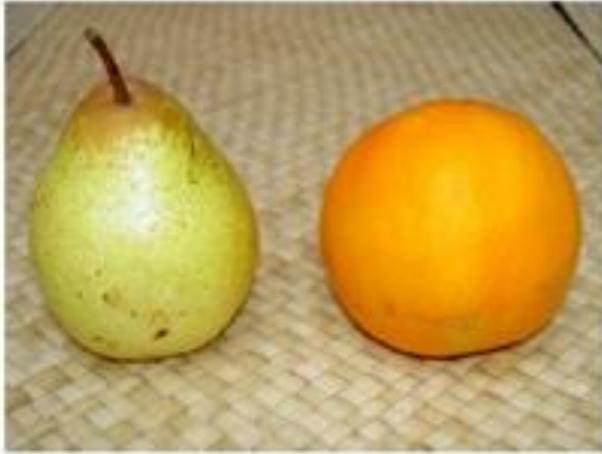
# Concealment



How would you do this with Poisson blending?

- Insert a copy of the background.

# Texture swapping



# References

Basic reading:

- Szeliski textbook, Sections 3.13, 3.5.5, 9.3.4, 10.4.3.

Additional reading:

- Pérez et al., “Poisson Image Editing,” SIGGRAPH 2003.  
the original Poisson blending paper.
- Georgiev, “Covariant Derivatives and Vision,” ECCV 2006.  
a paper from Adobe describing the version of Poisson blending implemented in Photoshop’s “healing brush”.
- Elder and Goldberg, “Image editing in the contour domain”, PAMI 2001.
- Bhat et al., “GradientShop: A Gradient-Domain Optimization Framework for Image and Video Filtering,” ToG 2010.
- Agrawal and Raskar, “Gradient Domain Manipulation Techniques in Vision and Graphics,” ICCV 2007 course, <http://www.amitkagrawal.com/ICCV2007Course/>  
the above references provide an overview of gradient-domain processing as a general image processing paradigm, which can be used for a broad set of applications beyond blending, including tone-mapping, colorization, converting to grayscale, edge enhancement, image abstraction and non-photorealistic rendering.