

# Subsampling and image pyramids



15-463, 15-663, 15-862  
Computational Photography  
Fall 2017, Lecture 5

# Course announcements

Respond to Doodle about rescheduling the September 27<sup>th</sup> lecture!

- Link available on Piazza.
- Currently 11 responses. Only one more since Monday :-( .

# Course announcements

- Homework 1 is due tomorrow.
  - Any remaining questions?
  - How was it?
- Homework 2 will be posted tonight and will be due two weeks from now.
  - Much larger than homework 1.
  - Start early! Experiments take a long time to run.

# Overview of today's lecture

- Finish non-linear filtering: non-local means.
- Image downsampling.
- Aliasing.
- Gaussian image pyramid.
- Laplacian image pyramid.

# Slide credits

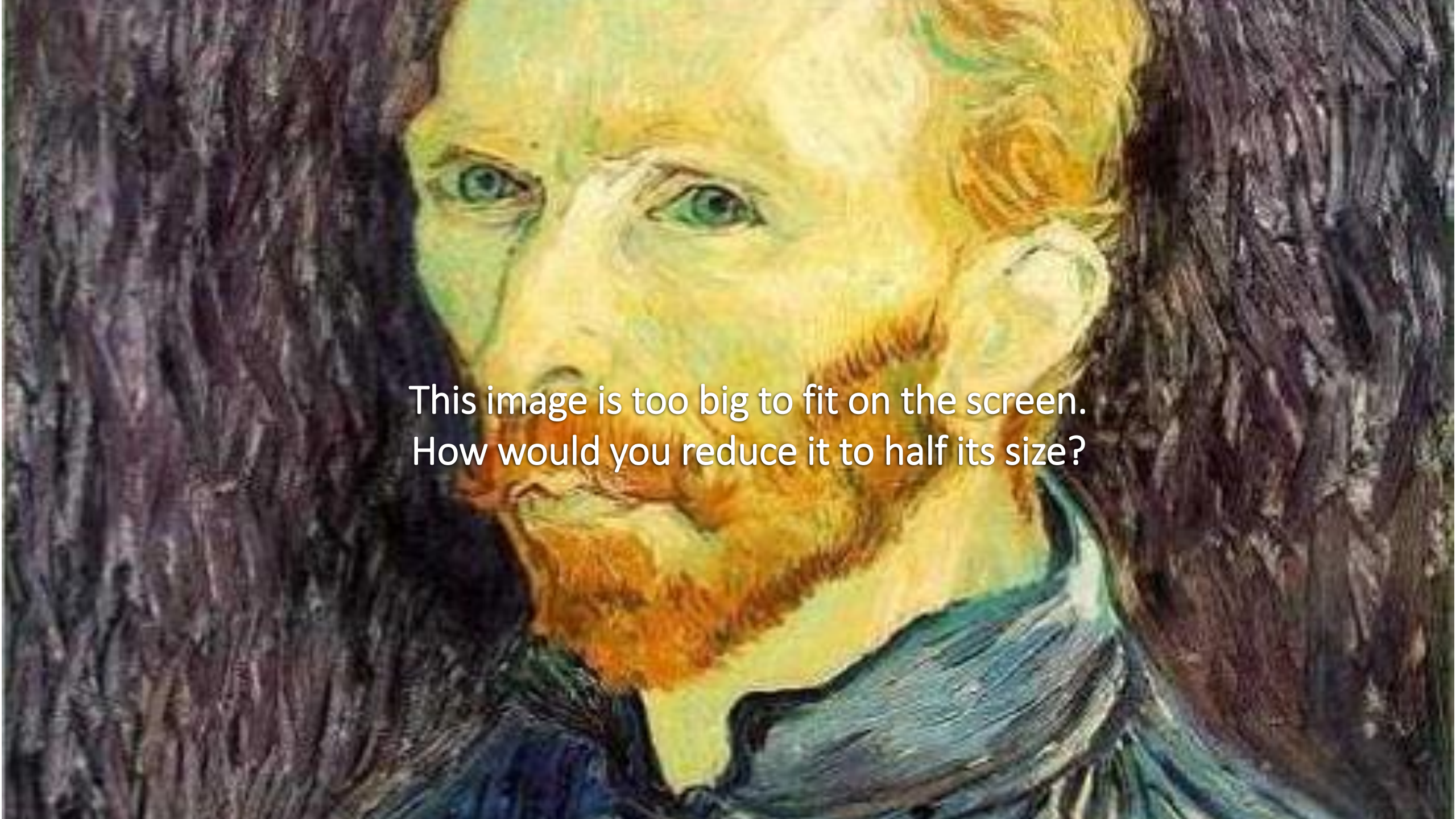
Most of these slides were adapted directly from:

- Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

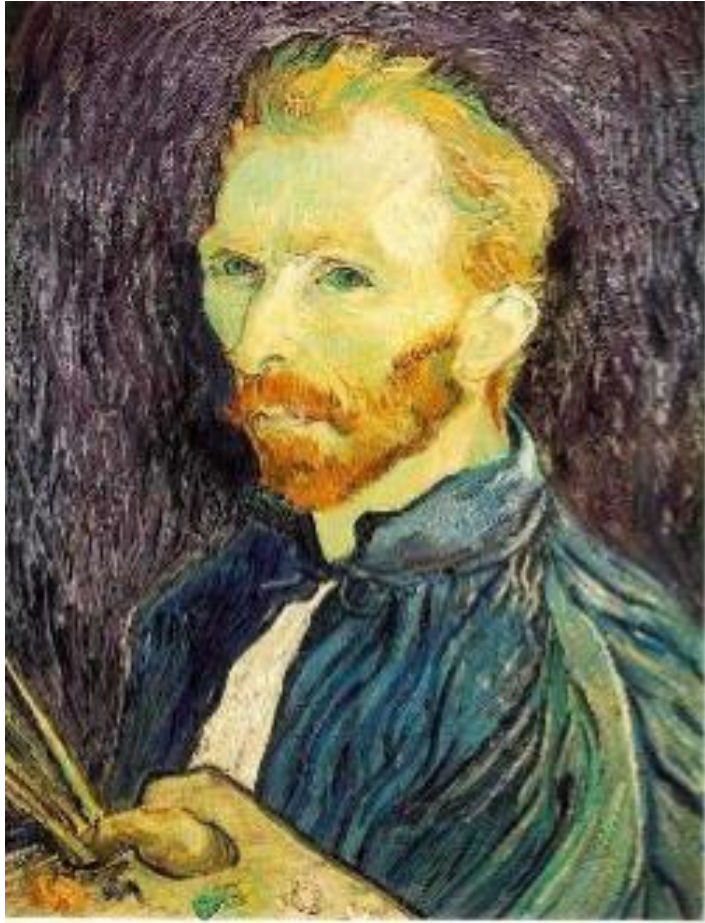
- Bernd Girod (Stanford University).
- Steve Marschner (Cornell University).
- Steve Seitz (University of Washington).

Image downsampling



This image is too big to fit on the screen.  
How would you reduce it to half its size?

# Naïve image downsampling



1/2

Throw away half the rows and columns

delete even rows  
delete even columns



1/4

delete even rows  
delete even columns

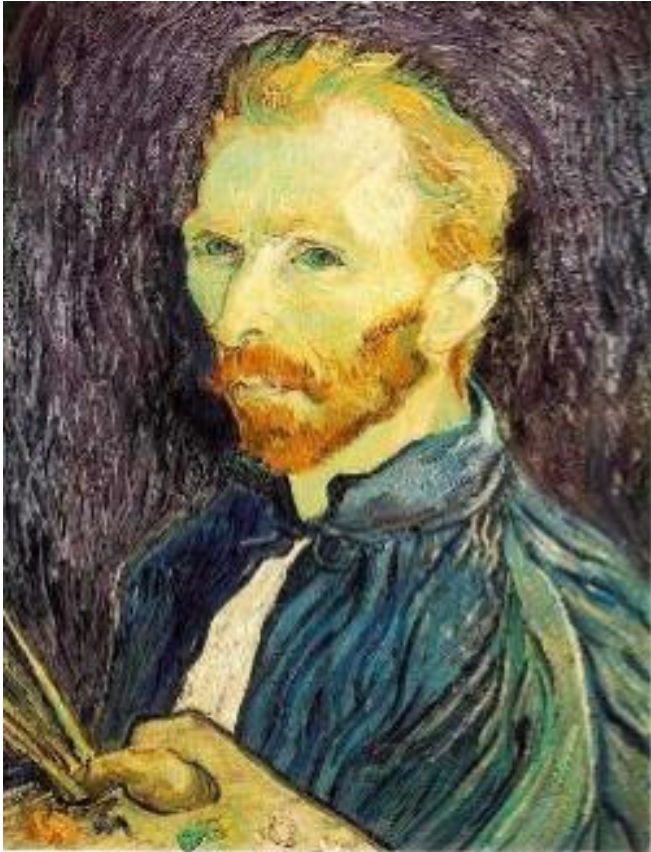


1/8

What is the problem with this approach?



# Naïve image downsampling



1/2



1/4 (2x zoom)

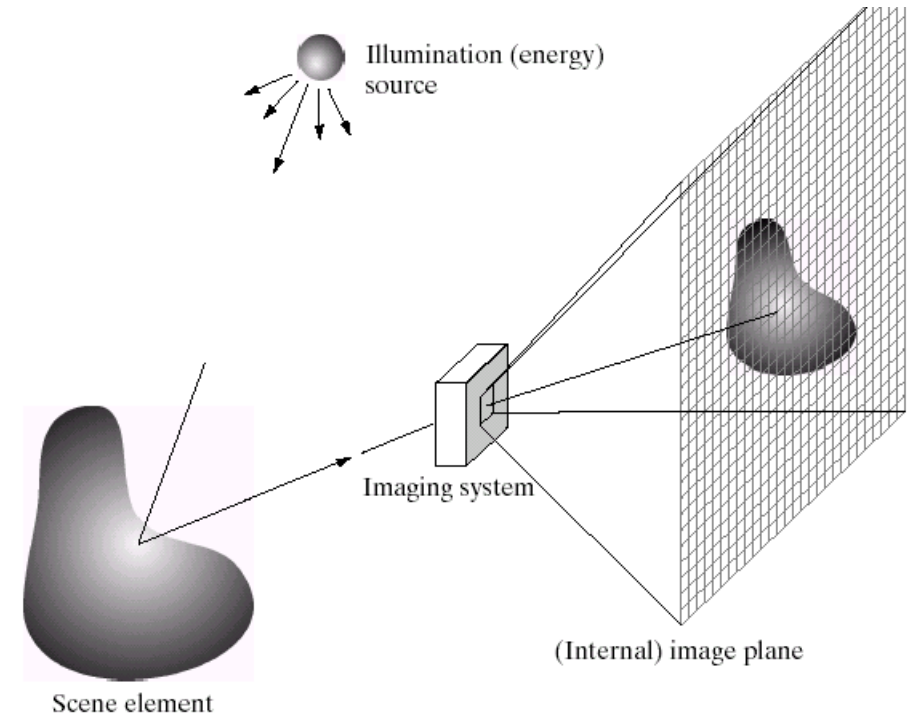


1/8 (4x zoom)

What is the 1/8 image so pixelated (and do you know what this effect is called)?

Aliasing

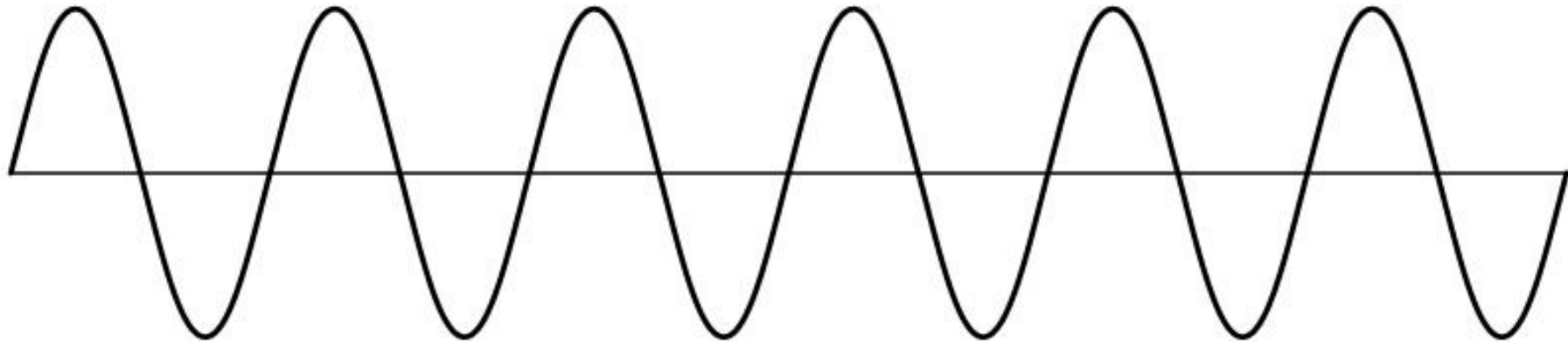
# Reminder



Images are a *discrete*, or *sampled*, representation of a *continuous* world

# Sampling

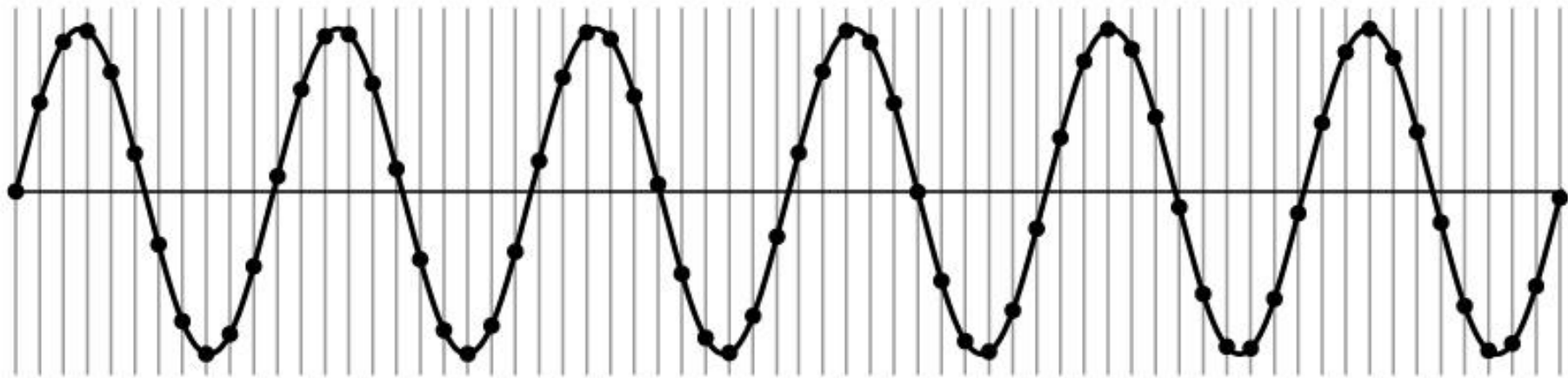
Very simple example: a sine wave



How would you discretize this signal?

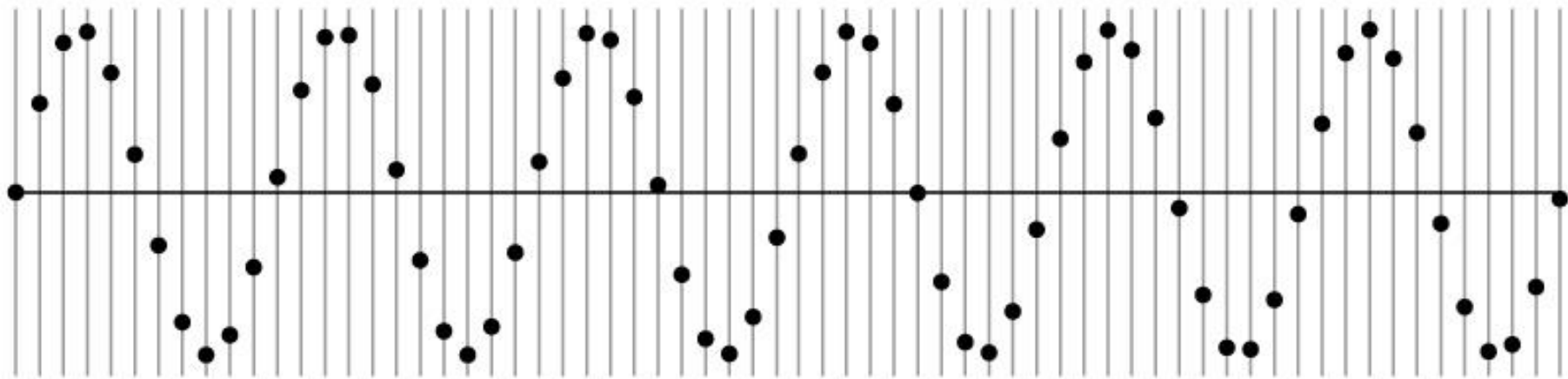
# Sampling

Very simple example: a sine wave



# Sampling

Very simple example: a sine wave

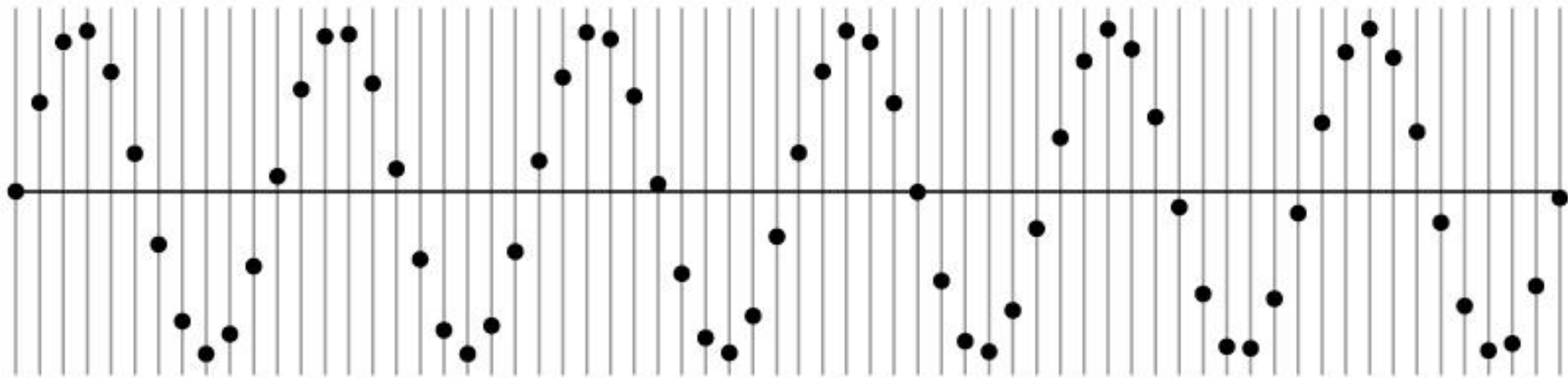


How many samples should I take?

Can I take as *many* samples as I want?

# Sampling

Very simple example: a sine wave

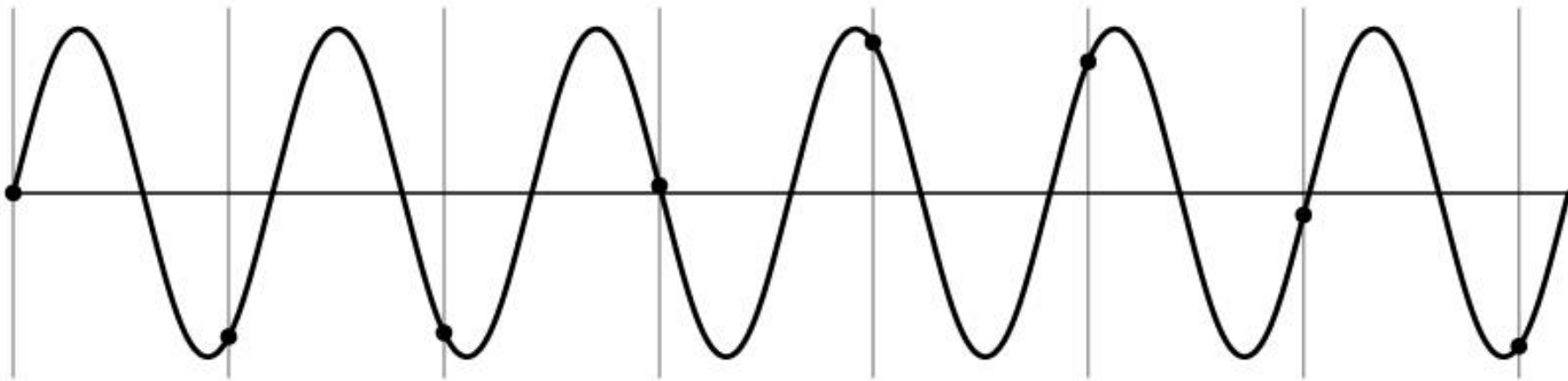


How many samples should I take?

Can I take as *few* samples as I want?

# Undersampling

Very simple example: a sine wave

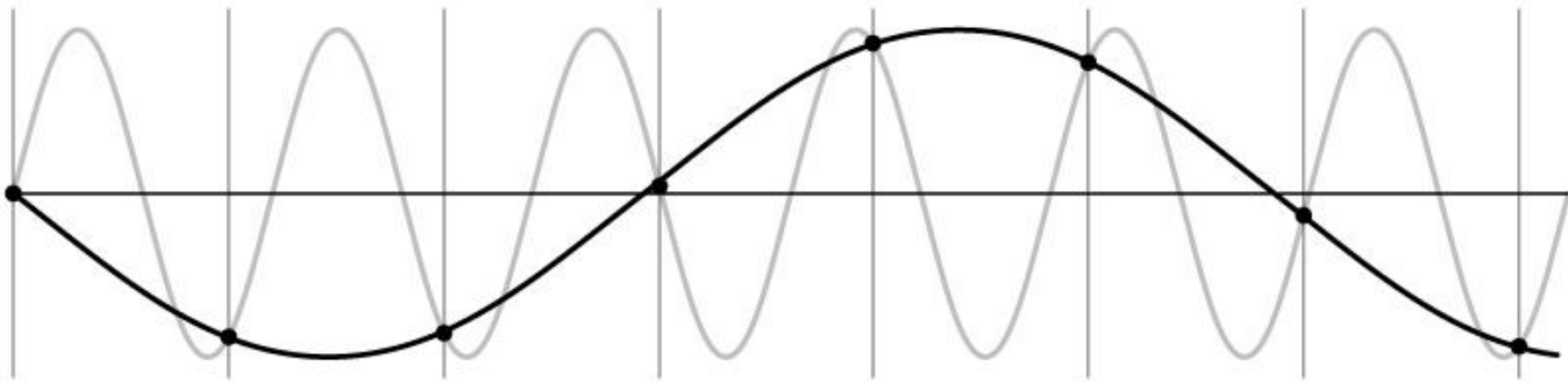


Unsurprising effect: information is lost.



# Undersampling

Very simple example: a sine wave

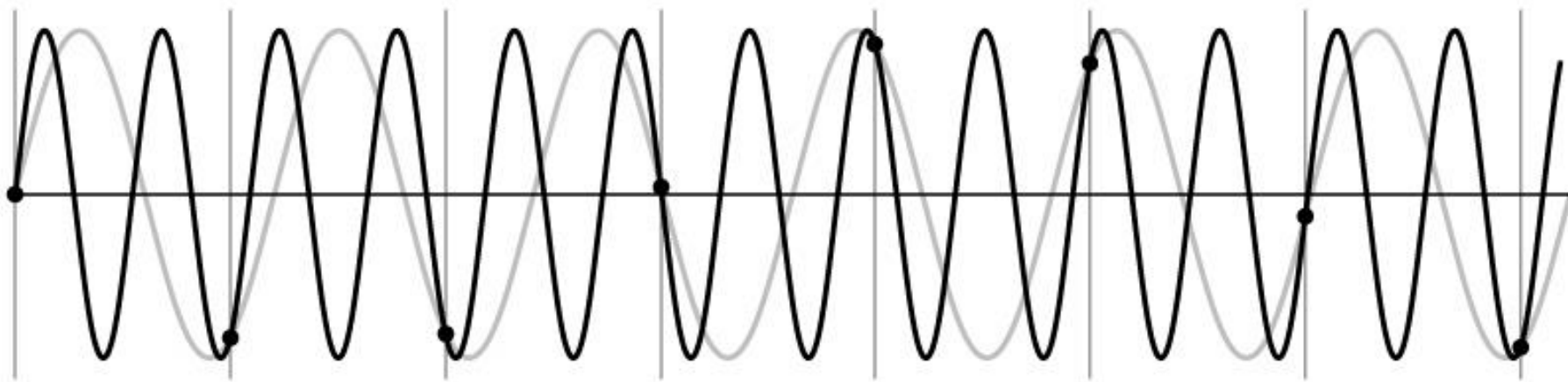


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

# Undersampling

Very simple example: a sine wave



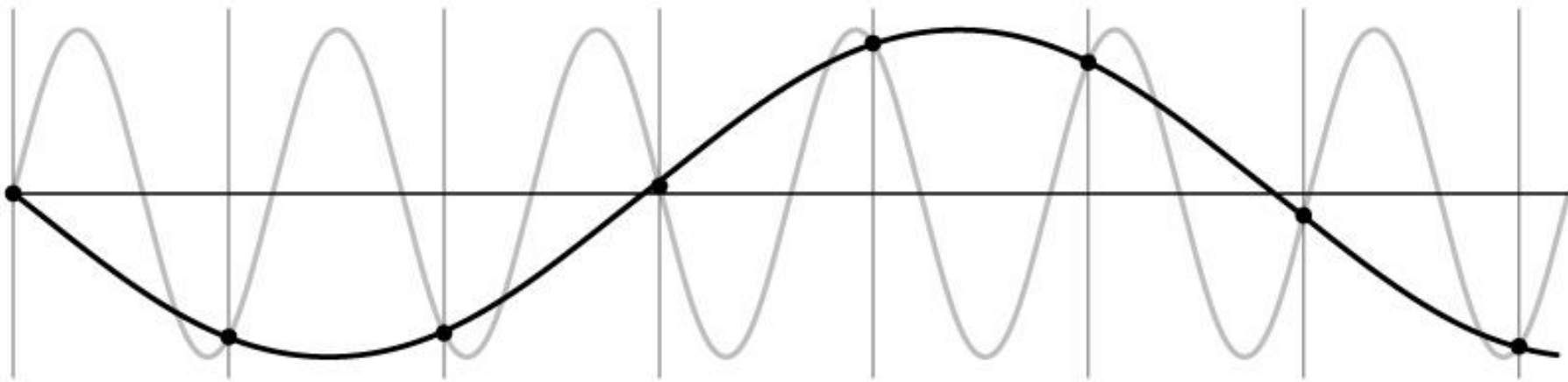
Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.

# Aliasing

Fancy term for: *Undersampling can disguise a signal as one of a lower frequency*

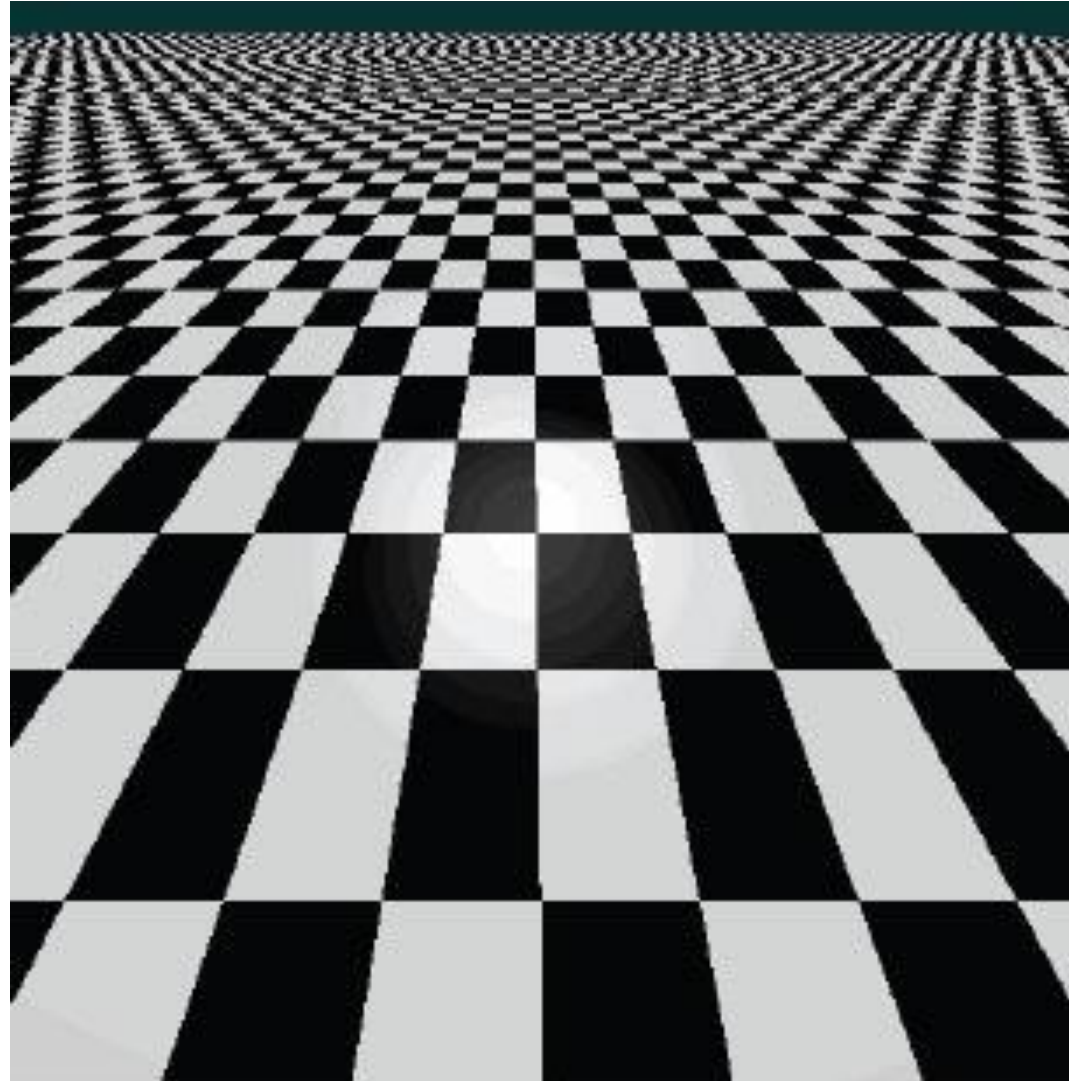


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.

# Aliasing in textures



# Aliasing in photographs

This is also known as “moire”

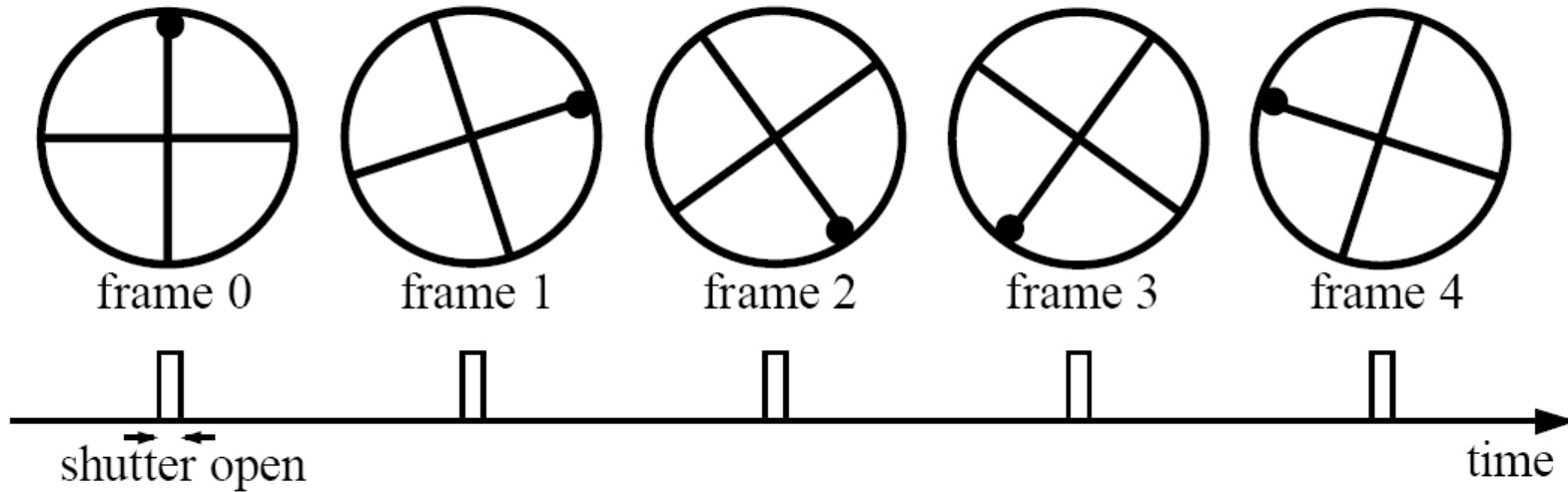


# Temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)

# Wagon wheel effect







# Anti-aliasing

How would you deal with aliasing?

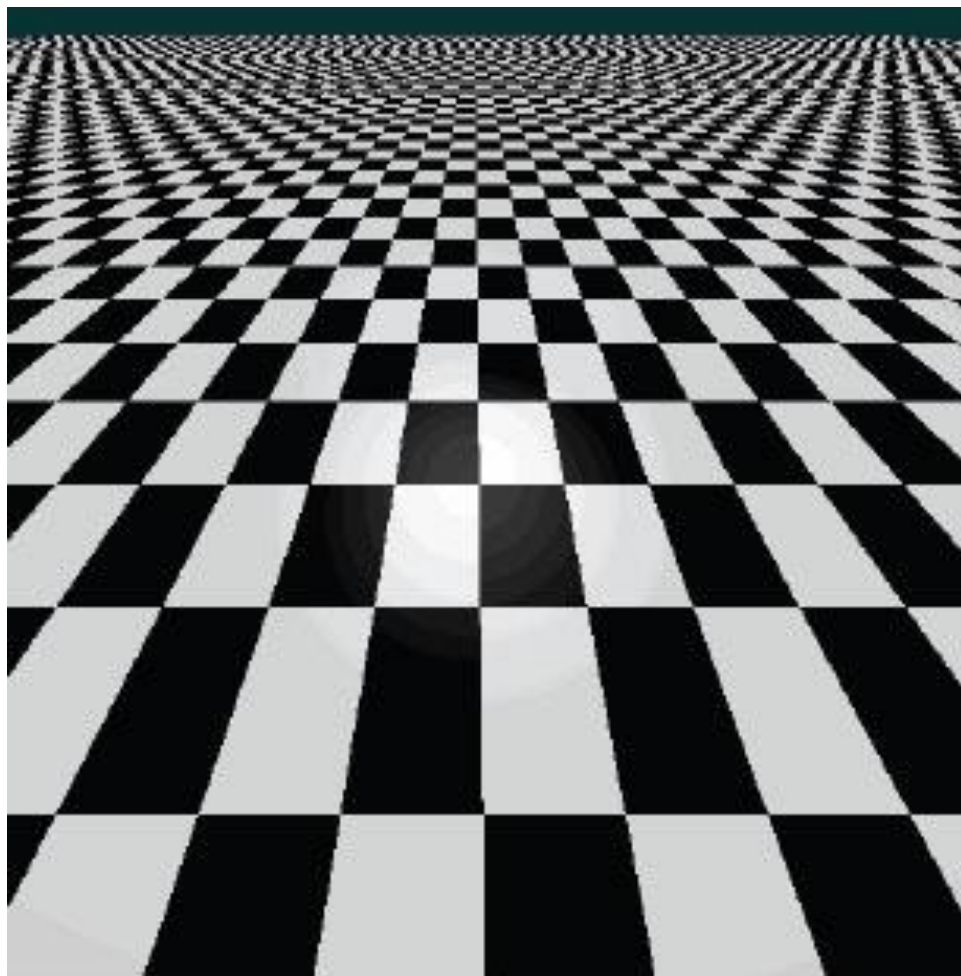
# Anti-aliasing

How would you deal with aliasing?

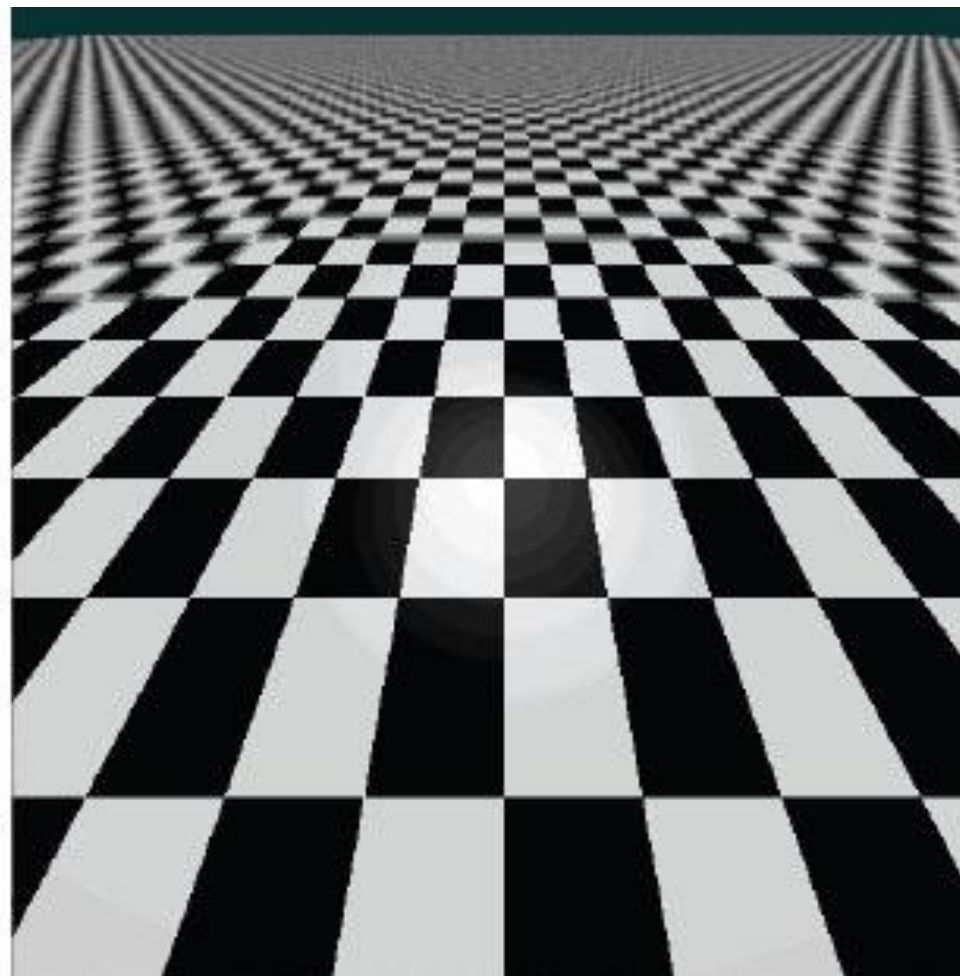
Approach 1: Oversample the signal

- This is how camera manufacturers started focusing so heavily on number of megapixels.

# Anti-aliasing in textures



aliasing artifacts



anti-aliasing by oversampling

# Anti-aliasing

How would you deal with aliasing?

Approach 1: Oversample the signal

- This is how camera manufacturers started focusing so heavily on number of megapixels.

Approach 2: Smooth the signal

- Remove some of the high frequency effects that cause aliasing.
- Lose information, but better than aliasing artifacts.

How would you smooth a signal?

# Anti-aliasing

Question 1: How much smoothing do I need to do to avoid aliasing?

Question 2: How many samples do I need to take to avoid aliasing?

Answer to both: Enough to reach the Nyquist limit.

We'll see what this means in the next lecture.

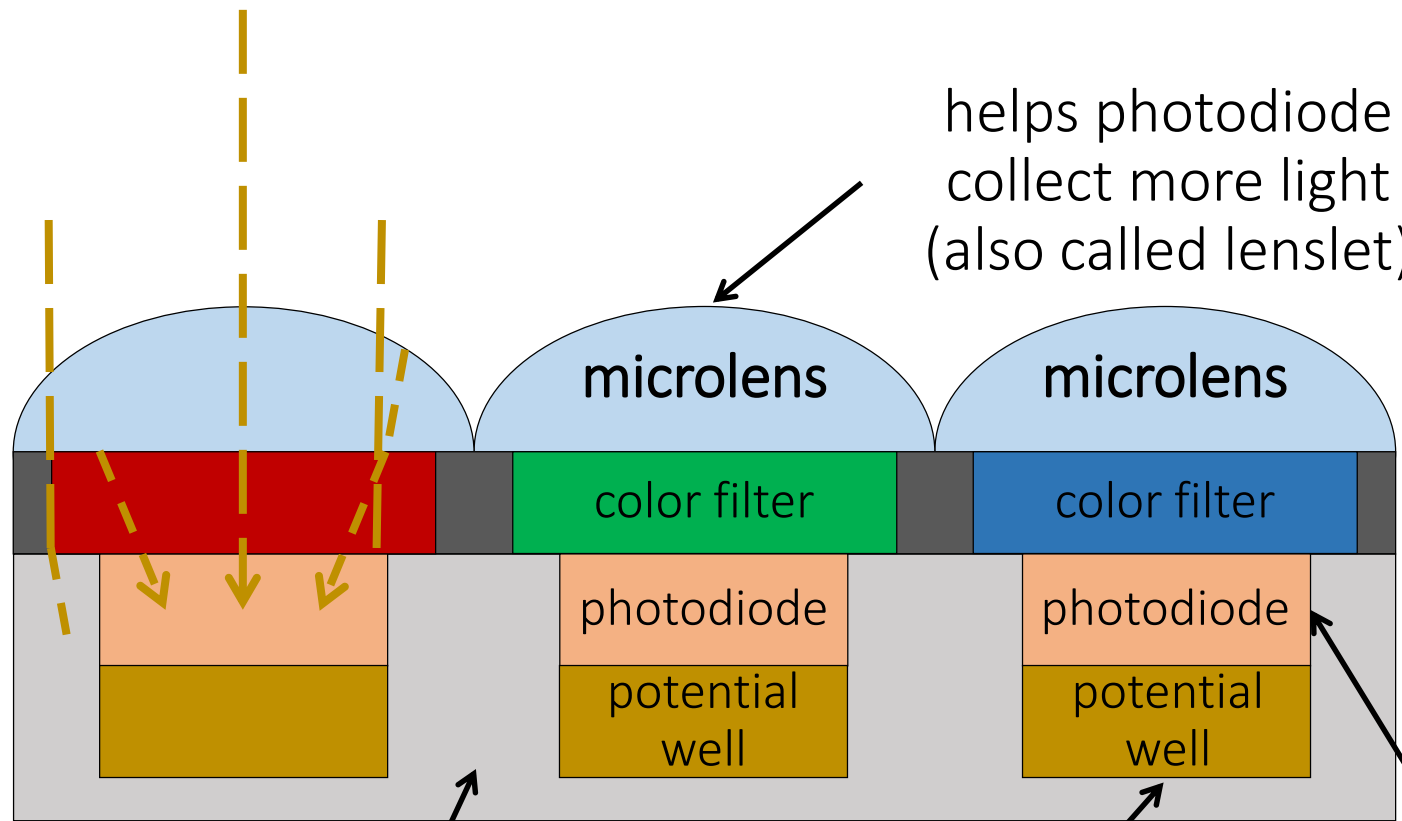
# Quick aside: optical anti-aliasing

Lenses act as (optical) smoothing filters.

# Quick aside: optical anti-aliasing

Lenses act as (optical) smoothing filters.

Slide from lecture 2: Basic imaging sensor design



helps photodiode collect more light (also called lenslet)

- **Lenslets also filter the image to avoid resolution artifacts.**
- Lenslets are problematic when working with coherent light.
- Many modern cameras do not have lenslet arrays.

We will discuss these issues in more detail at a later lecture.

silicon for read-out etc. circuitry

stores emitted electrons

made of silicon, emits electrons from photons

# Quick aside: optical anti-aliasing

Lenses act as (optical) smoothing filters.

- Sensors often have a lenslet array in front of them as an anti-aliasing (AA) filter.
- However, the AA filter means you also lose resolution.
- Nowadays, due the large number of sensor pixels, AA filters are becoming unnecessary.



Photographers often hack their cameras to remove the AA filter, in order to avoid the loss of resolution.

a.k.a. “hot rodding”



# Quick aside: optical anti-aliasing

Example where AA filter is needed



without AA filter



with AA filter

# Quick aside: optical anti-aliasing

Example where AA filter is unnecessary



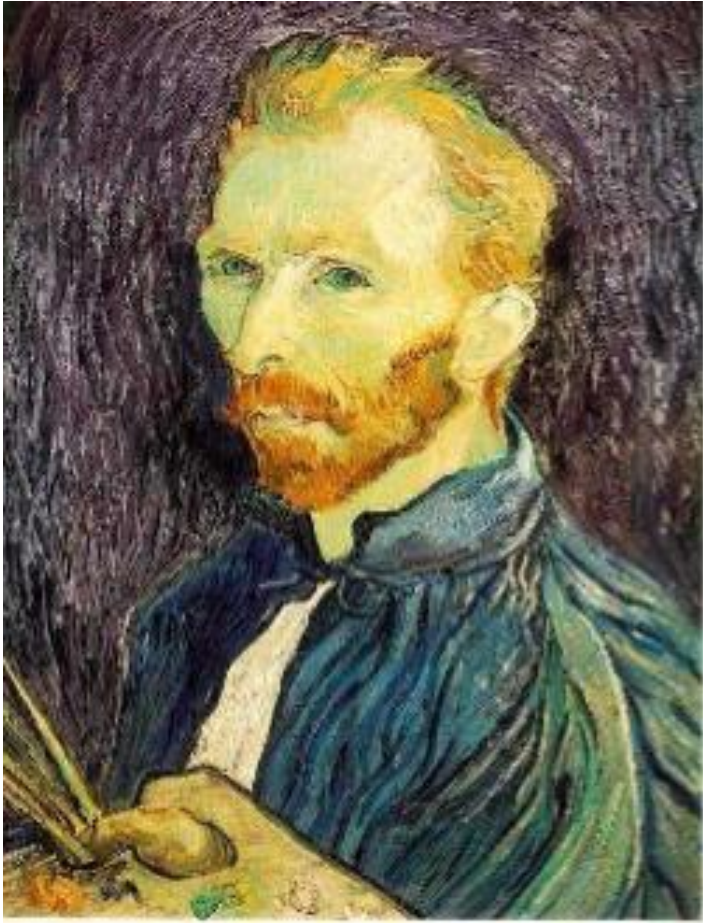
without AA filter



with AA filter

# Better image downsampling

Apply a smoothing filter first, then throw away half the rows and columns



1/2

Gaussian filter  
delete even rows  
delete even columns



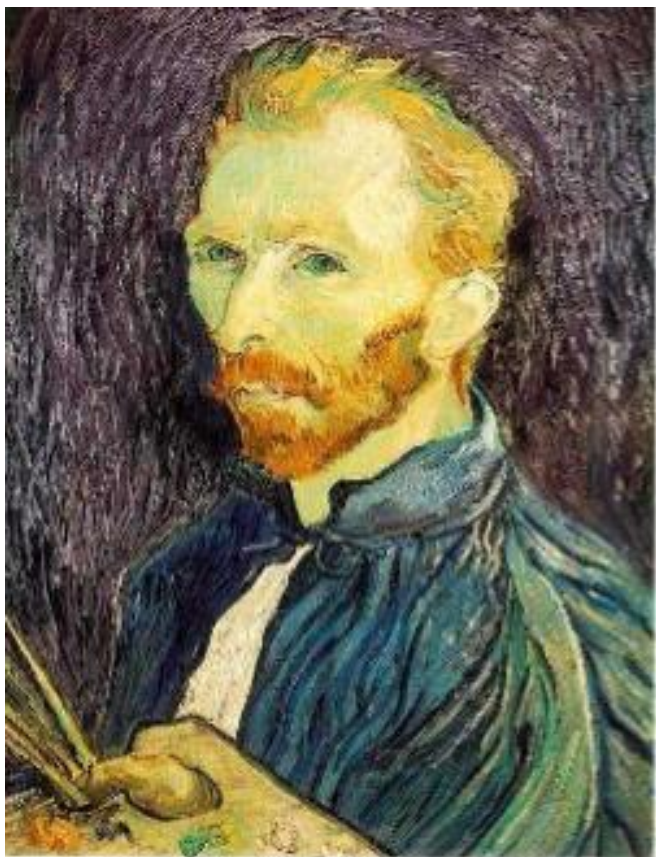
1/4

Gaussian filter  
delete even rows  
delete even columns



1/8

# Better image downsampling



1/2

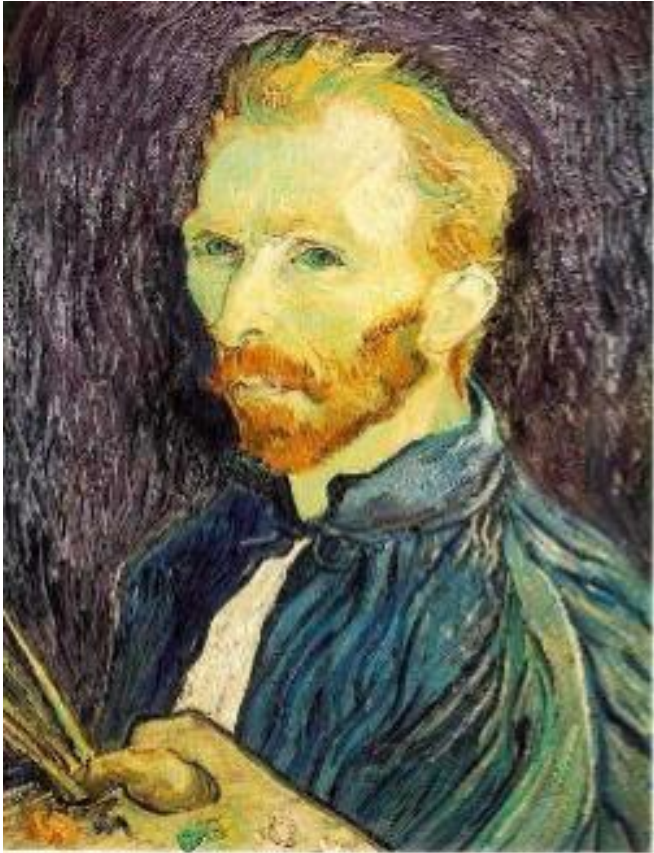


1/4 (2x zoom)



1/8 (4x zoom)

# Naïve image downsampling



1/2



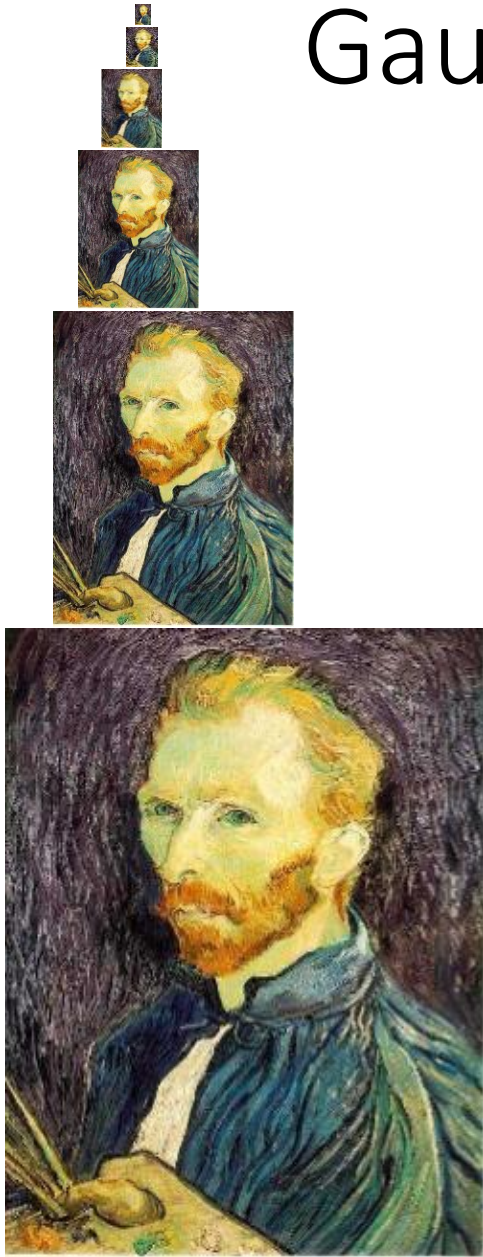
1/4 (2x zoom)



1/8 (4x zoom)

Gaussian image pyramid

# Gaussian image pyramid

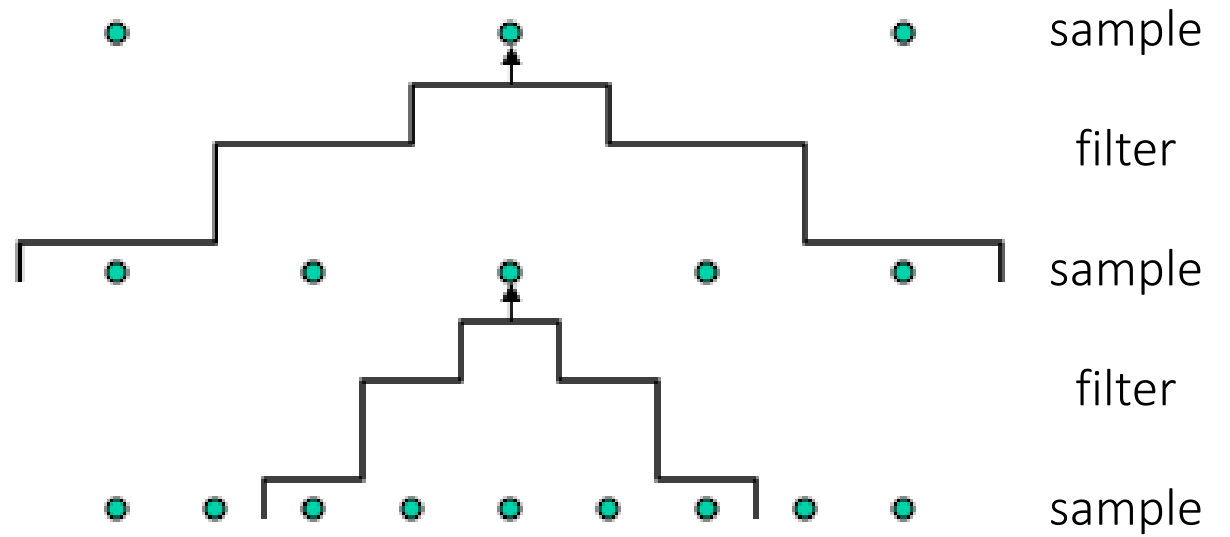


The name of this sequence of subsampled images

# Constructing a Gaussian pyramid

## Algorithm

```
repeat:  
  filter  
  subsample  
until min resolution reached
```



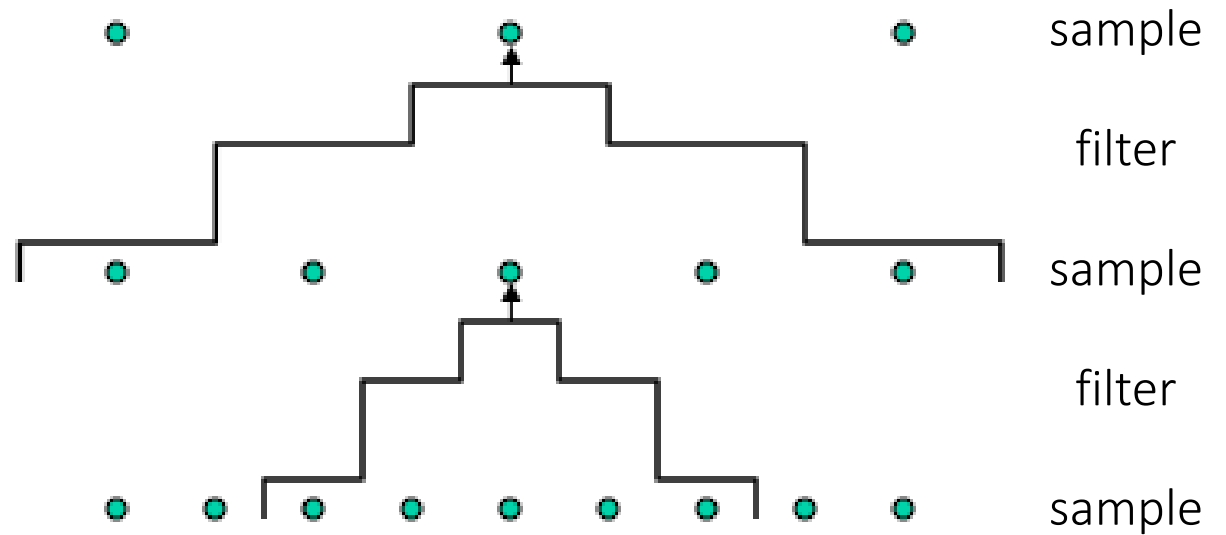
Question: How much bigger than the original image is the whole pyramid?



# Constructing a Gaussian pyramid

## Algorithm

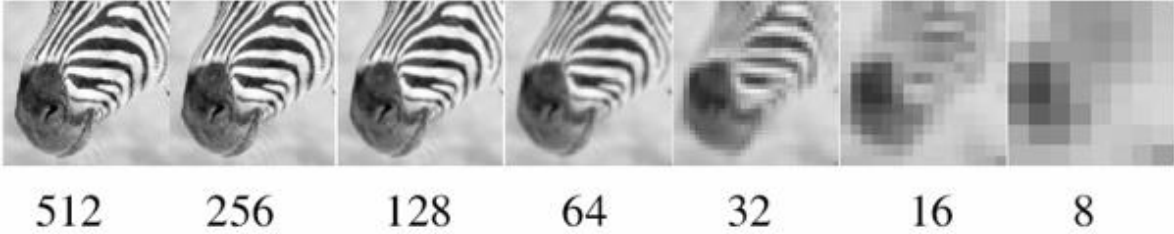
```
repeat:  
  filter  
  subsample  
until min resolution reached
```



Question: How much bigger than the original image is the whole pyramid?

Answer: Just  $\frac{4}{3}$  times the size of the original image! (How did I come up with this number?)

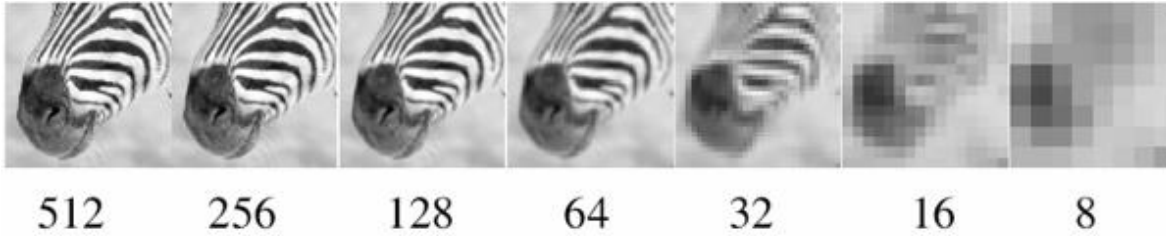
# Some properties of the Gaussian pyramid



What happens to the details of the image?



# Some properties of the Gaussian pyramid



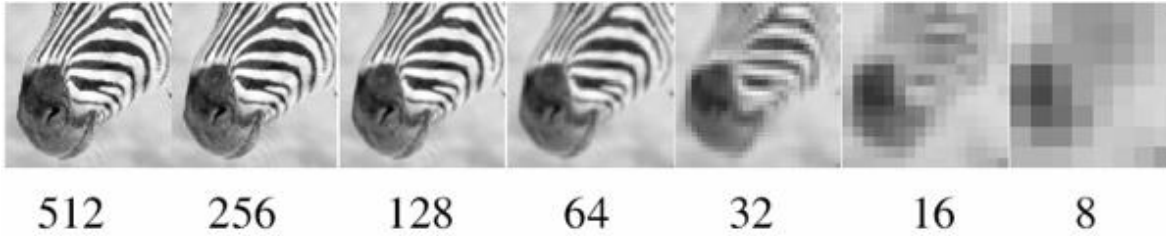
What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?



# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

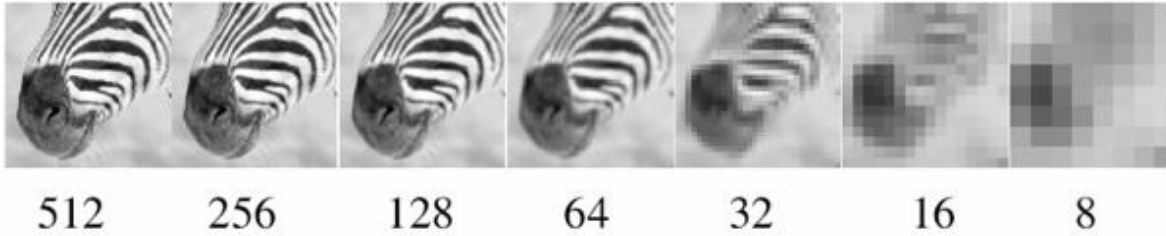
What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?



# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

- That's not possible.

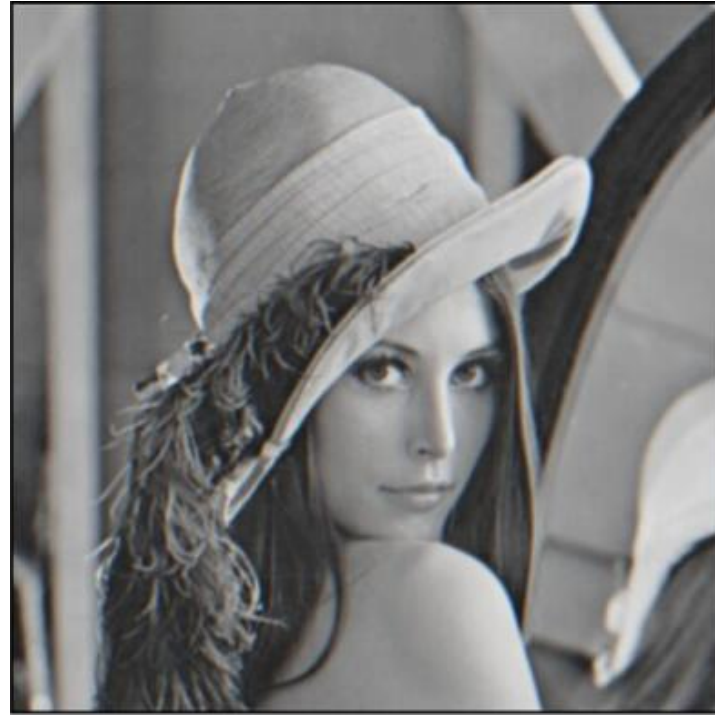


# Blurring is lossy



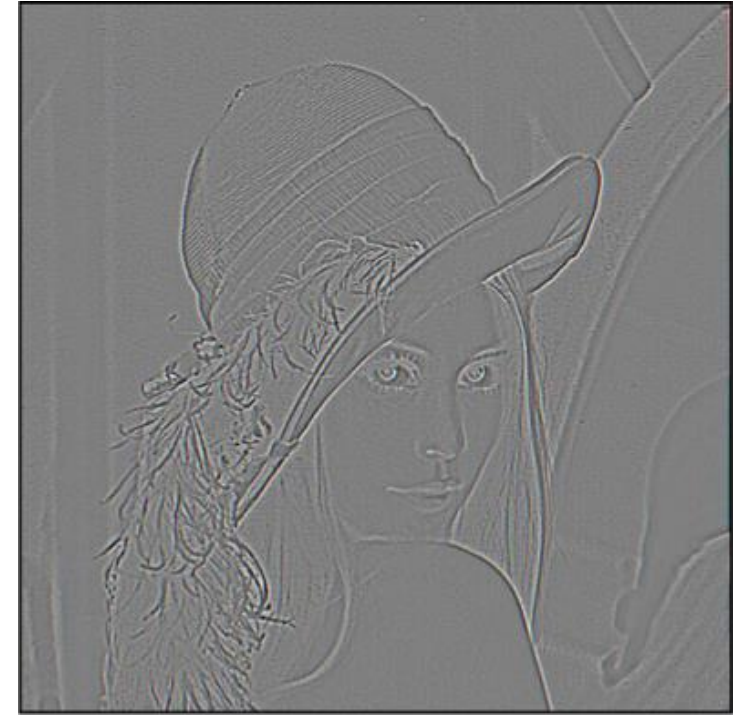
level 0

-



level 1 (before downsampling)

=



residual

What does the residual look like?

# Blurring is lossy



level 0

-



level 1 (before downsampling)

=



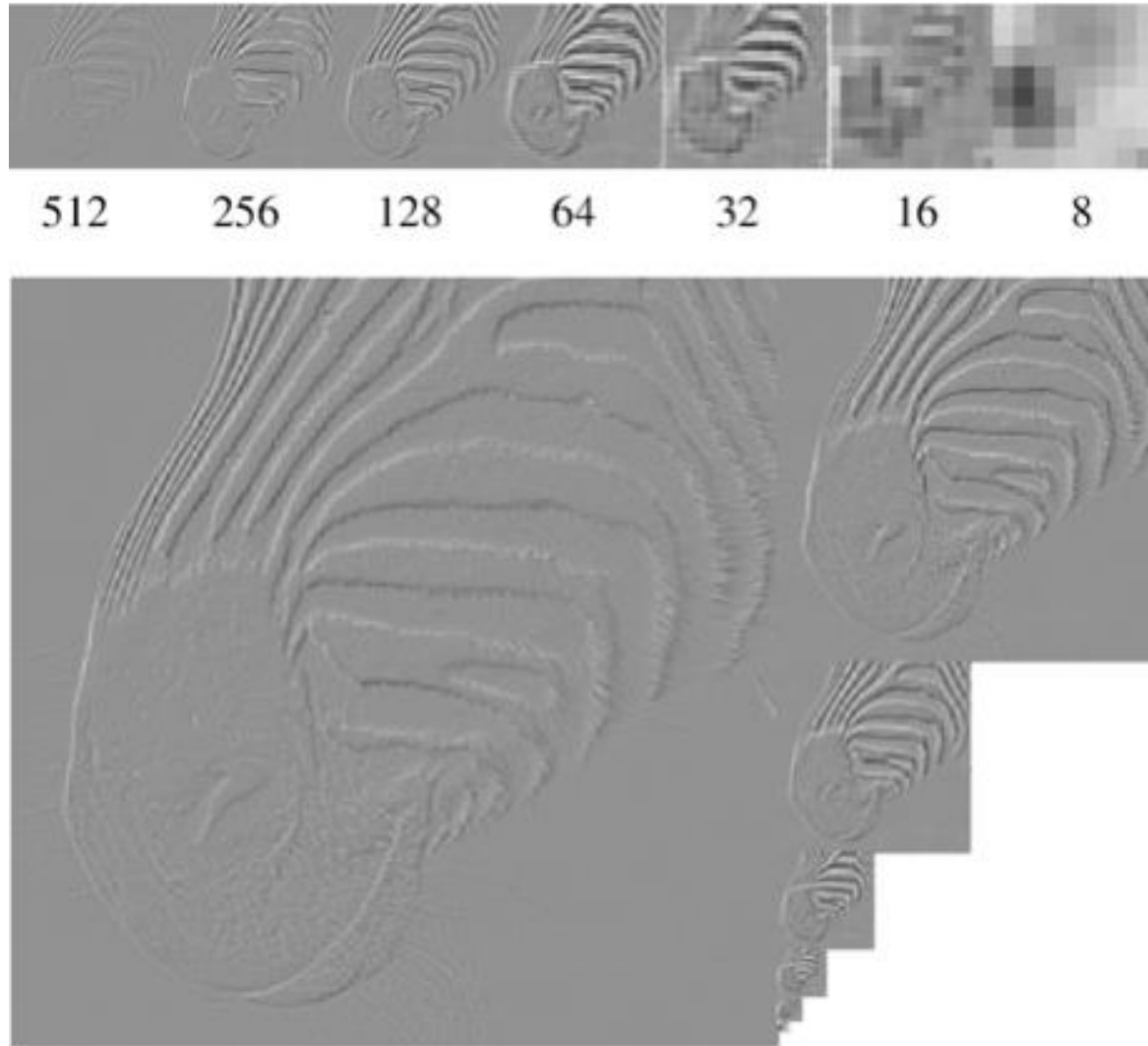
residual

Can we make a pyramid that is lossless?

Laplacian image pyramid



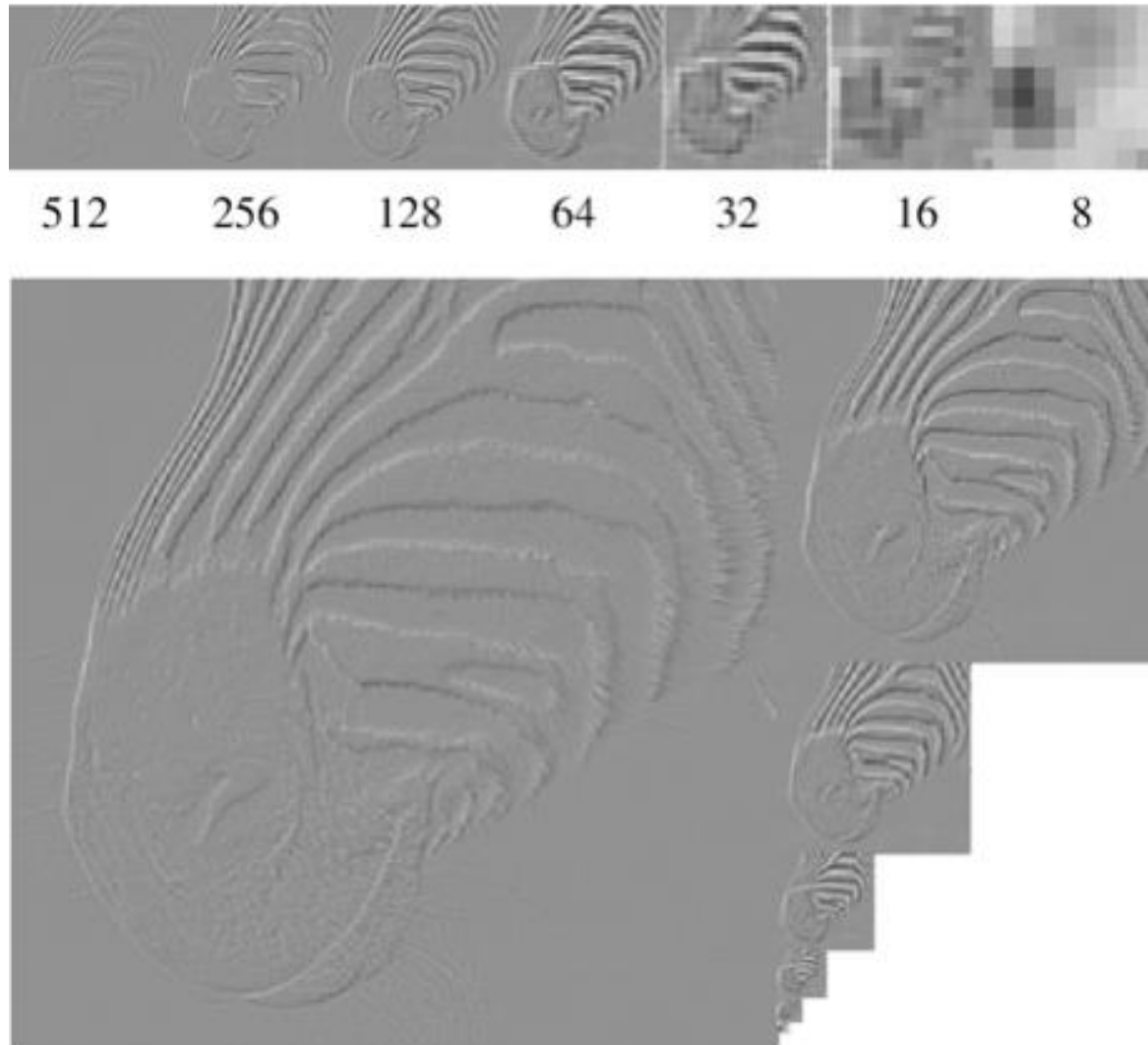
# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?

# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?

- Yes we can!



What do we need to store to be able to reconstruct the original image?

Let's start by looking at just one level



level 0

=



level 1 (upsampled)

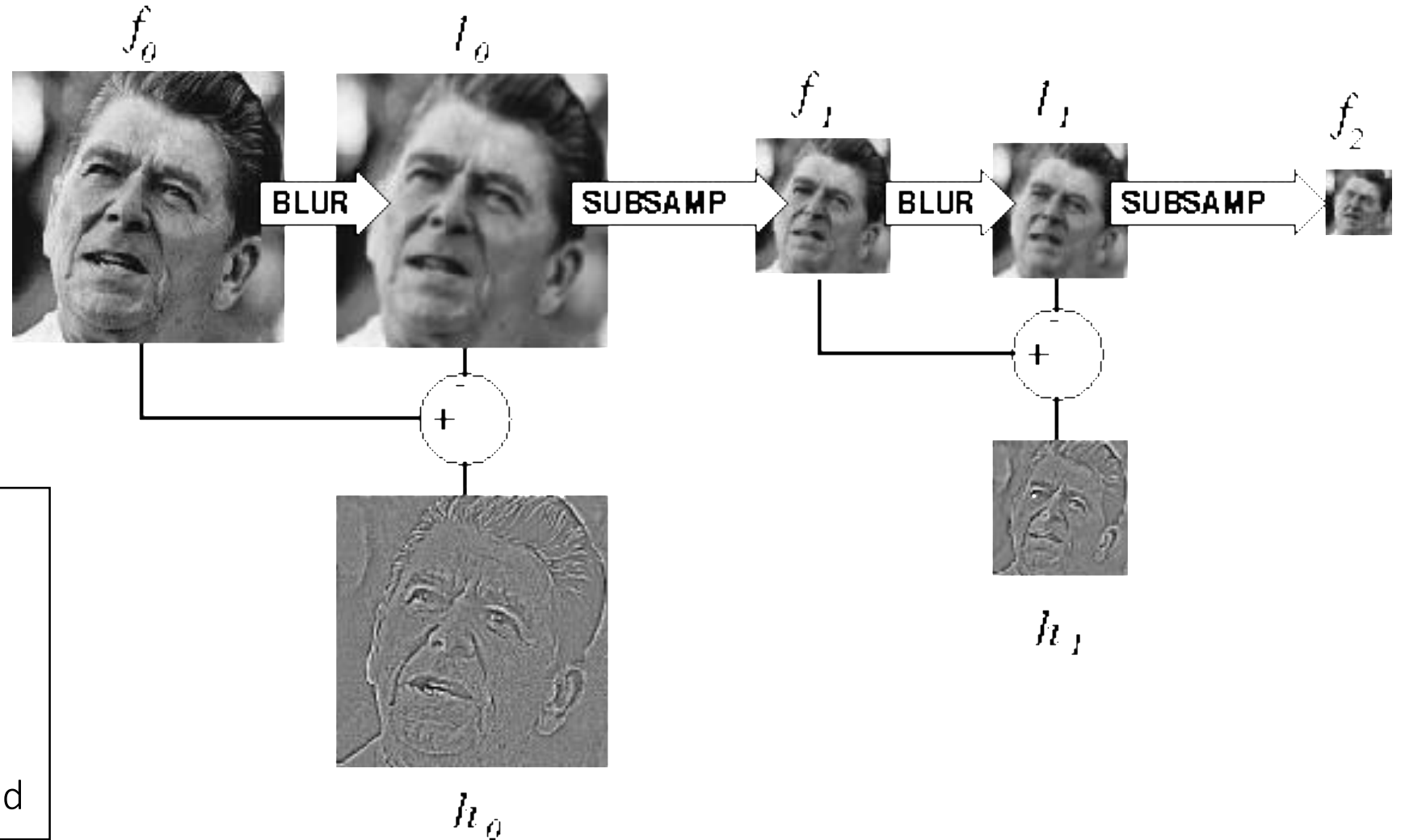
+



residual

Does this mean we need to store both residuals and the blurred copies of the original?

# Constructing a Laplacian pyramid



## Algorithm

repeat:

filter

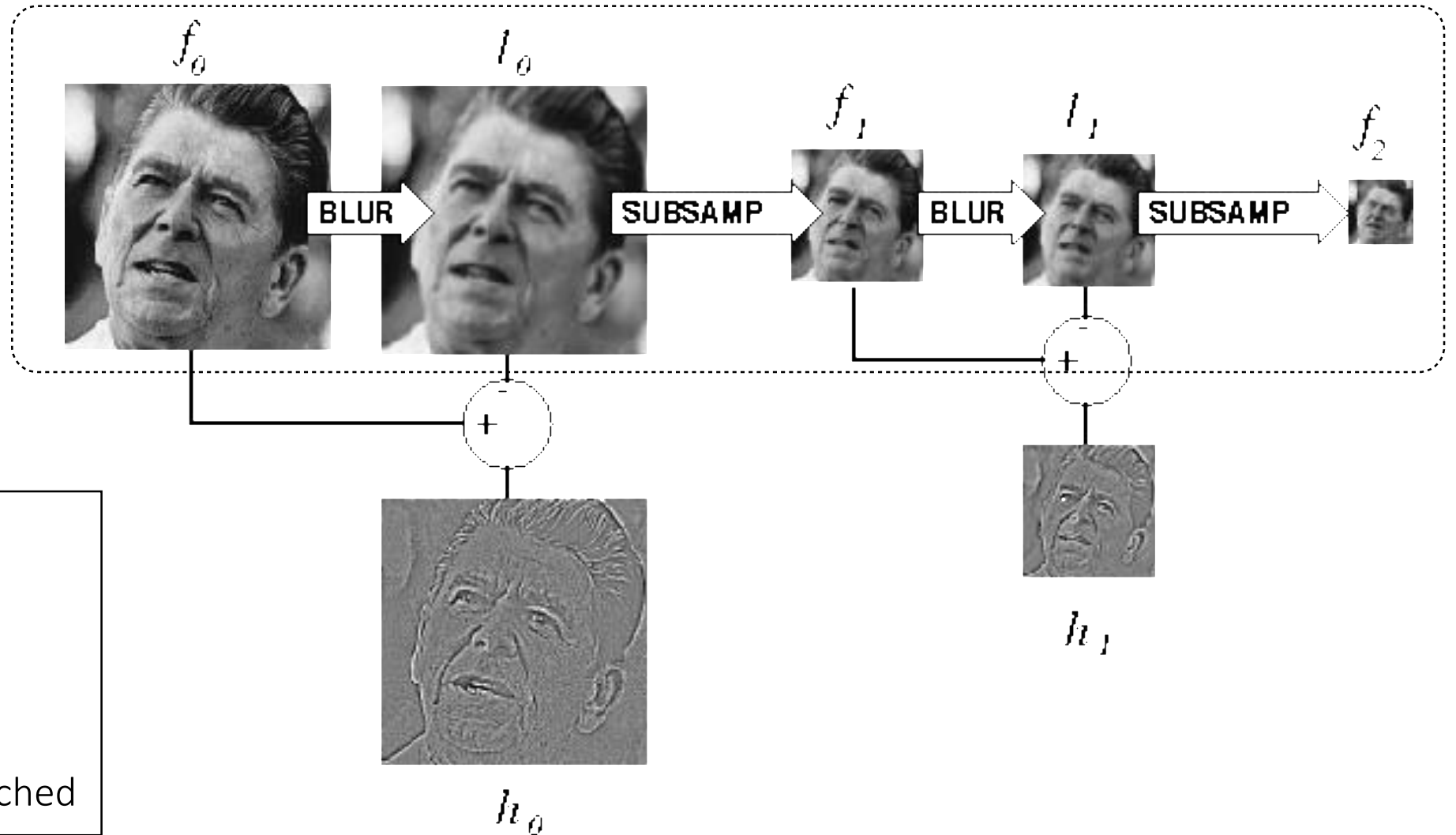
compute residual

subsample

until min resolution reached

# Constructing a Laplacian pyramid

What is this part?



## Algorithm

repeat:

filter

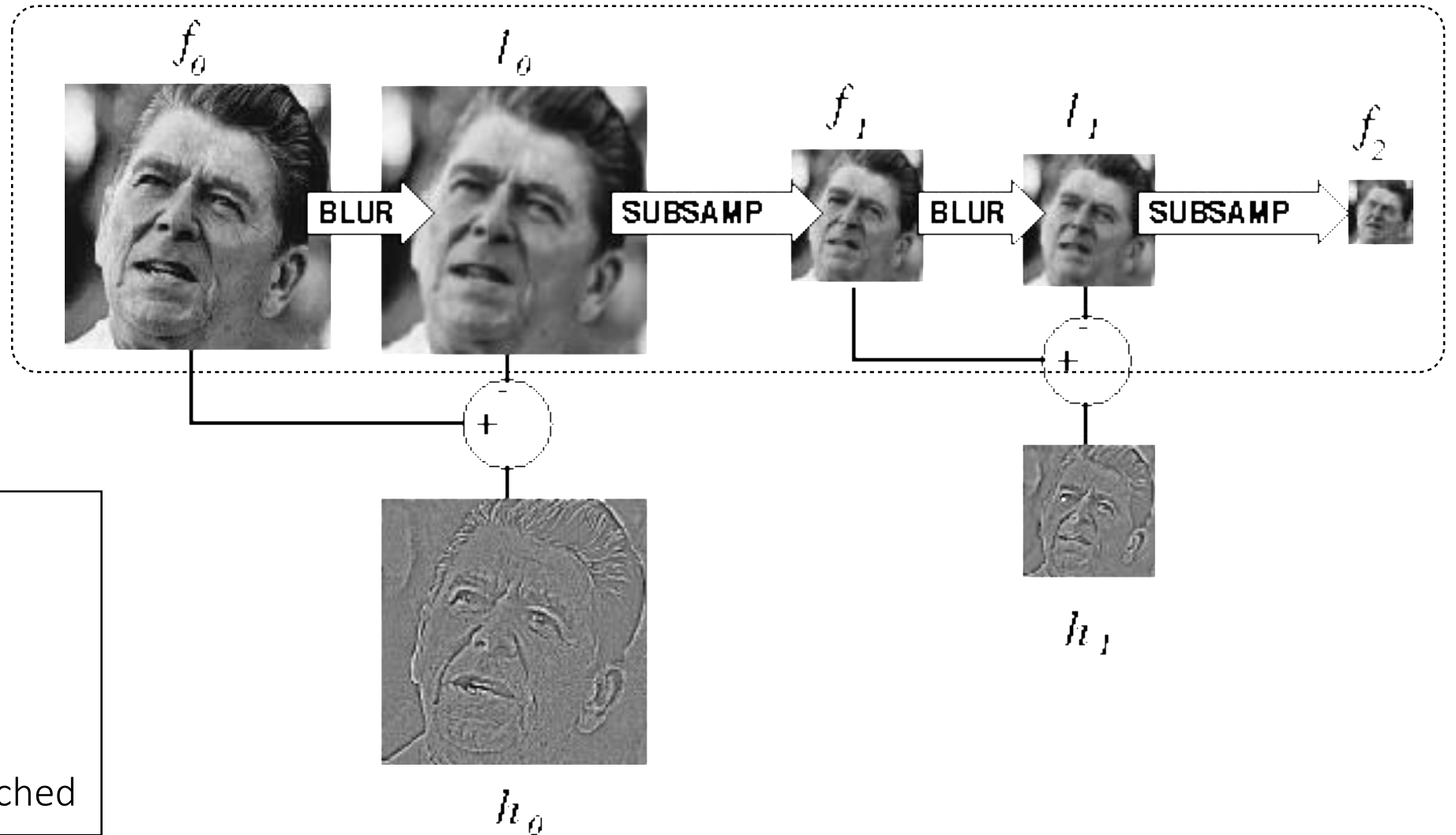
compute residual

subsample

until min resolution reached

# Constructing a Laplacian pyramid

What is this part?



## Algorithm

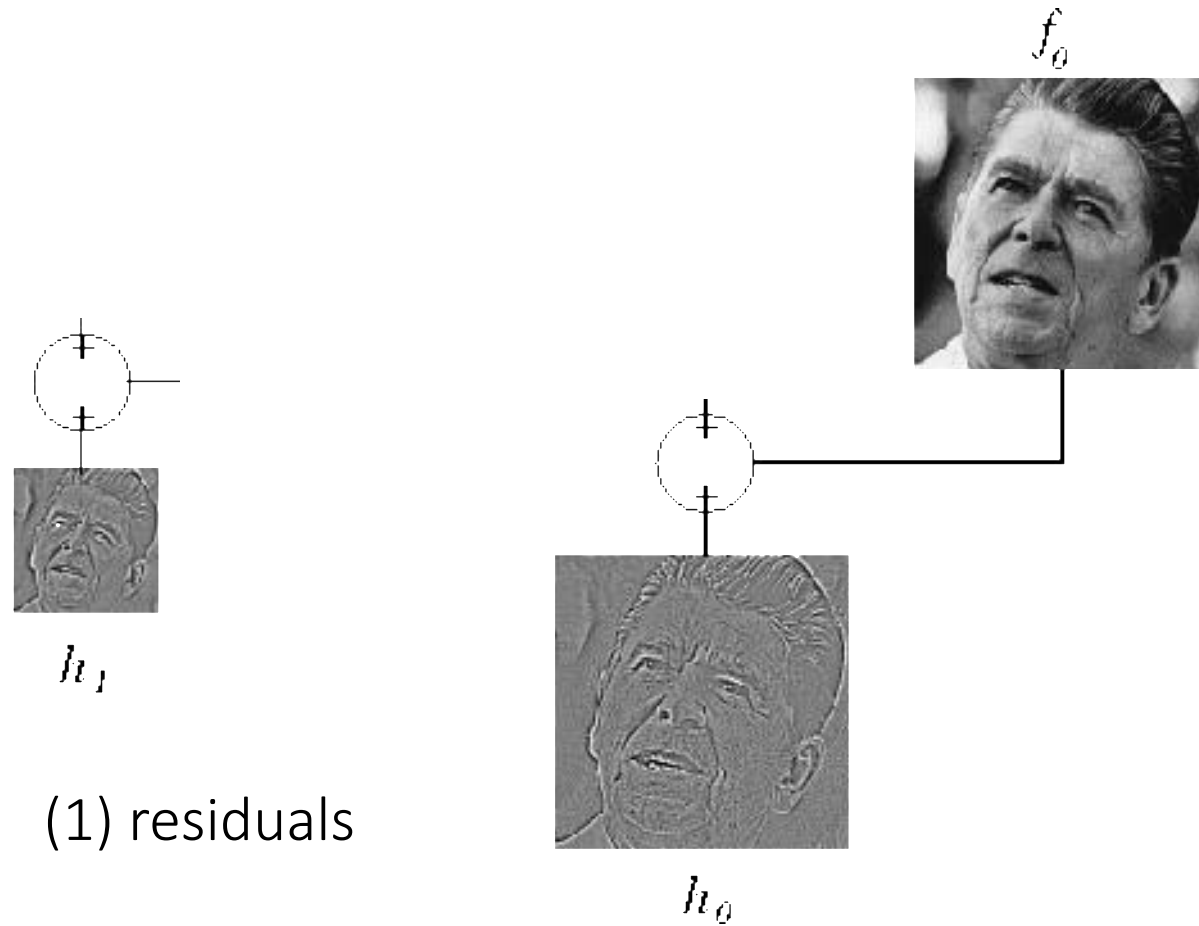
repeat:  
    filter  
    compute residual  
    subsample  
until min resolution reached

What do we need to construct the original image?

$f_0$



# What do we need to construct the original image?

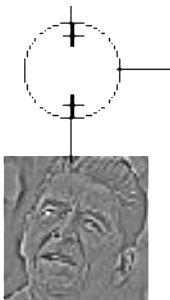


(1) residuals



# What do we need to construct the original image?

(2) smallest image  $f_2$

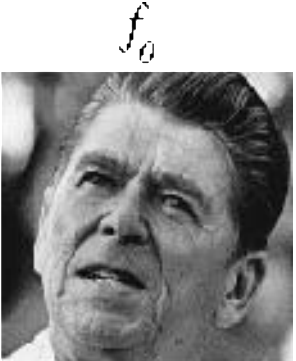


$h_1$

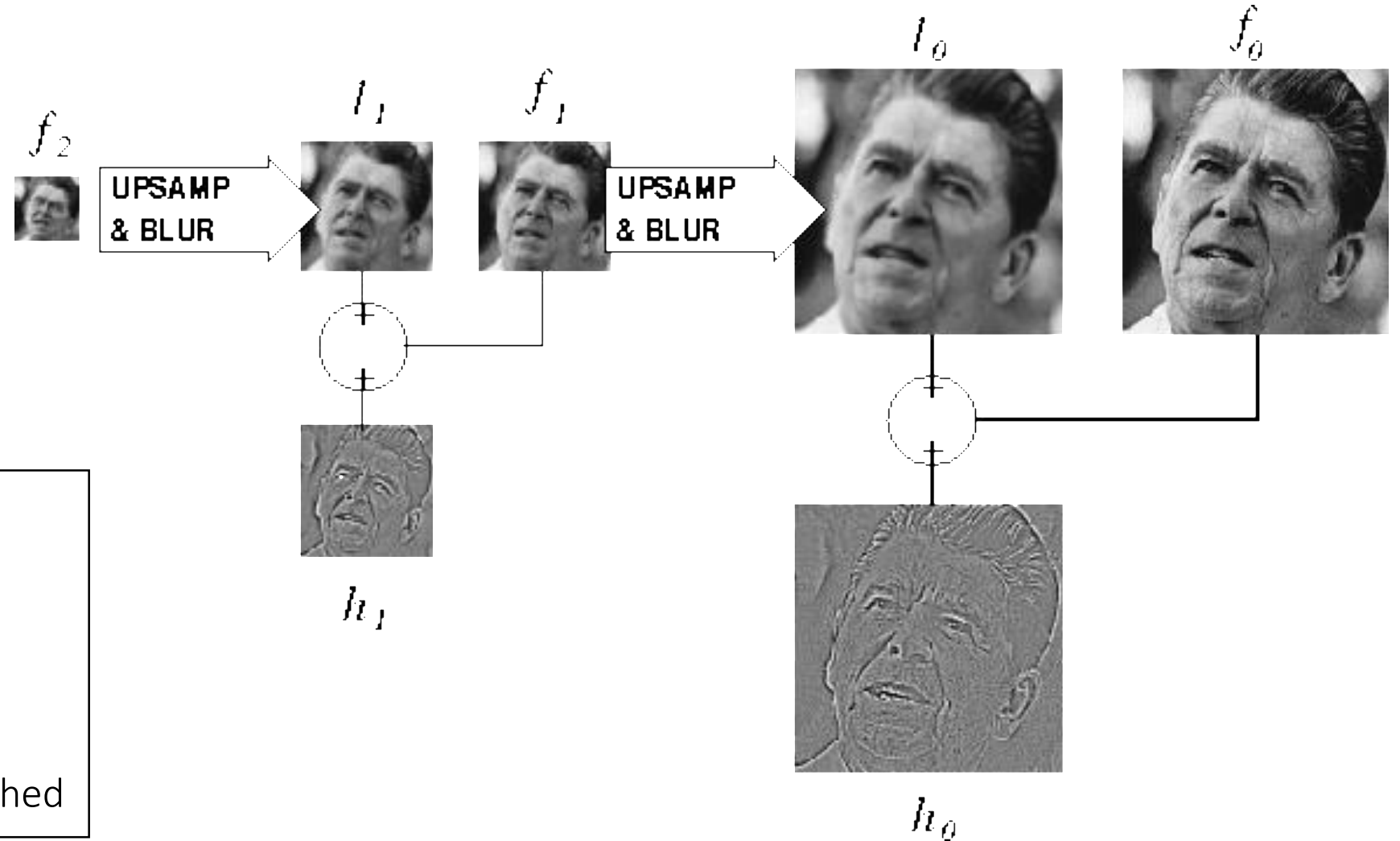
(1) residuals



$h_0$



# Reconstructing the original image



## Algorithm

repeat:

upsample

sum with residual

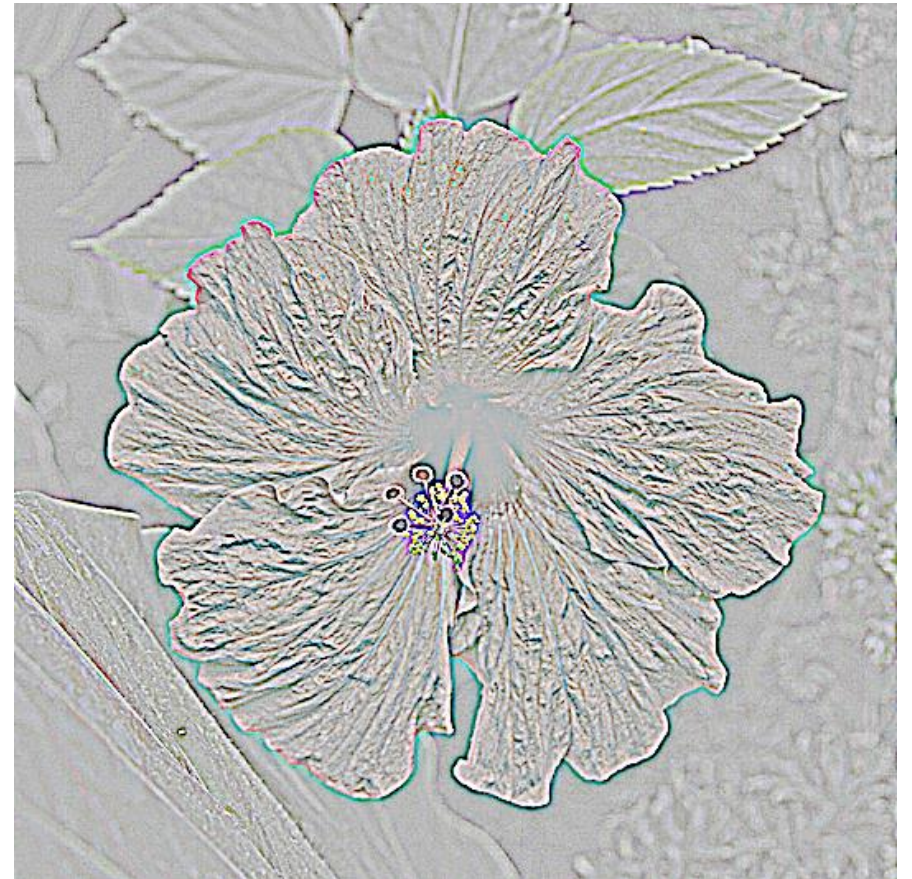
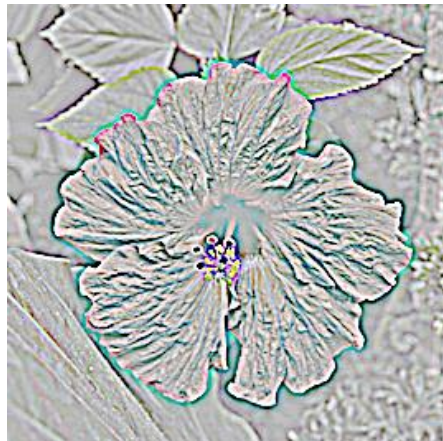
until orig resolution reached

# Gaussian vs Laplacian Pyramid



Shown in opposite order for space.

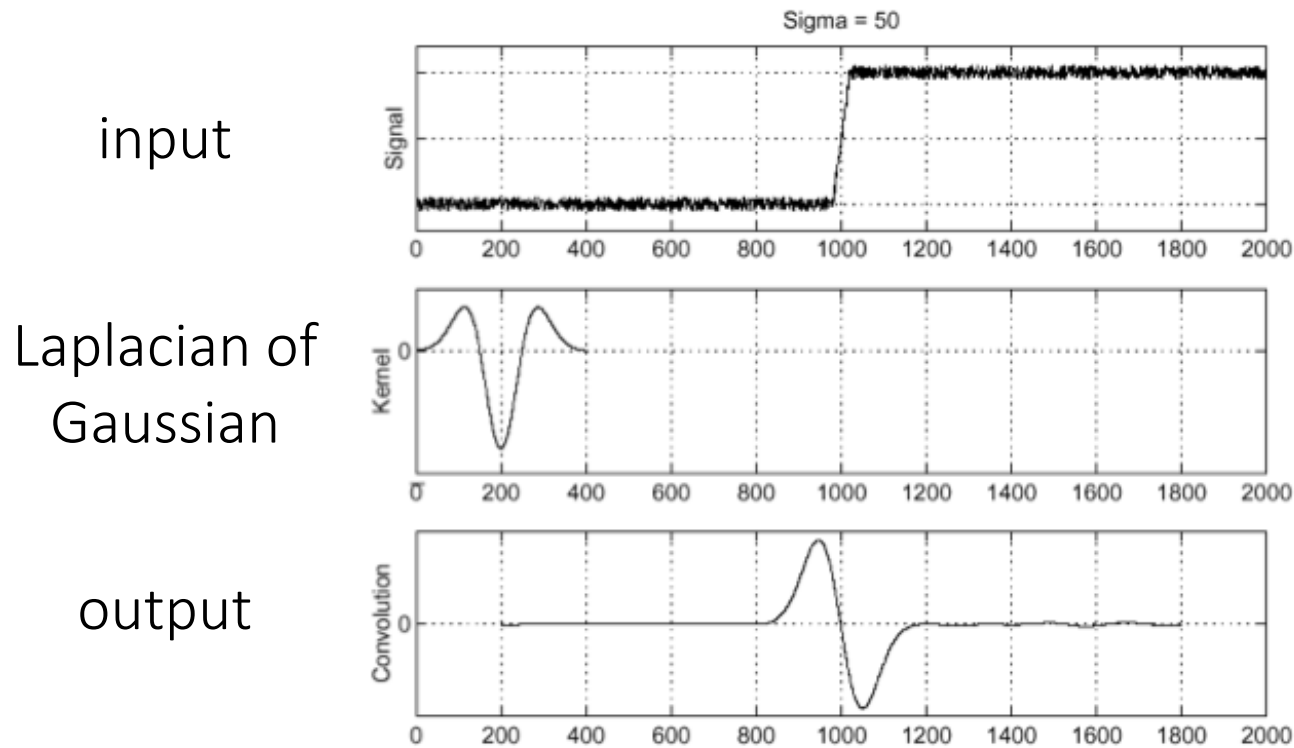
Which one takes more space to store?



Why is it called a Laplacian pyramid?

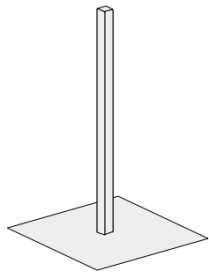
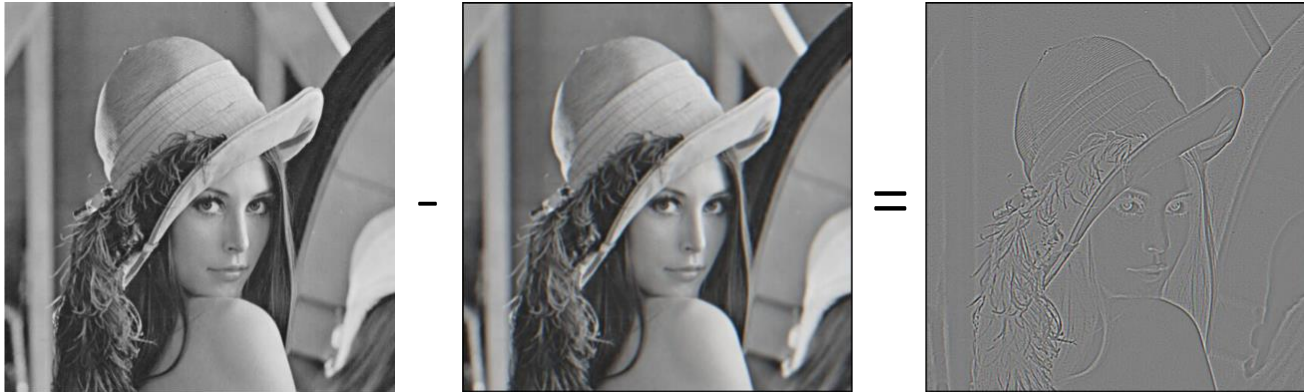
# Reminder: Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

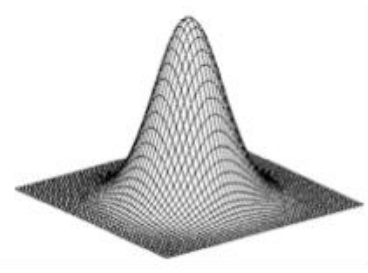


“zero crossings” at edges

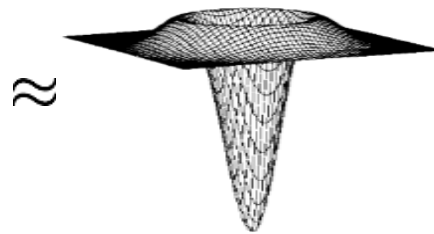
# Why is it called a Laplacian pyramid?



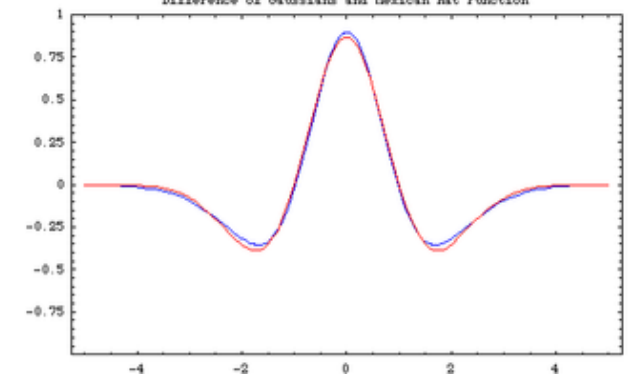
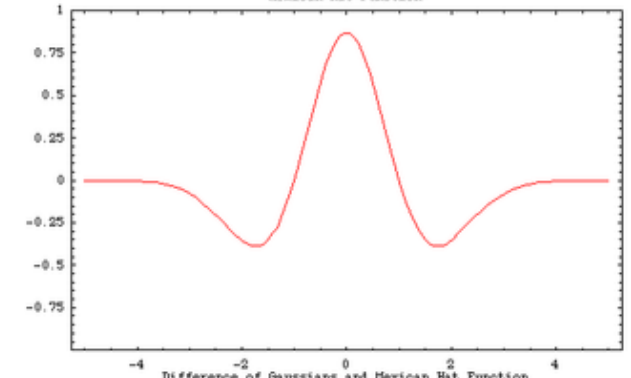
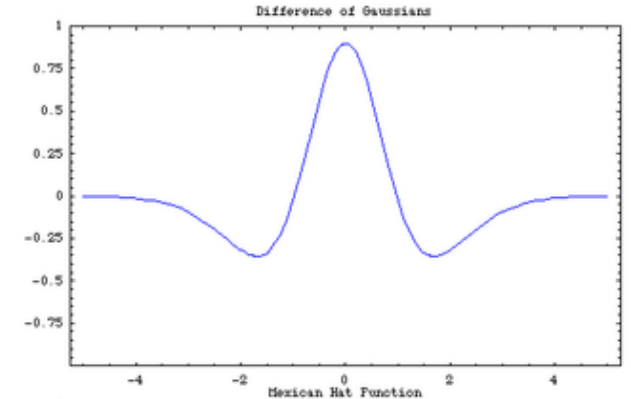
unit



Gaussian

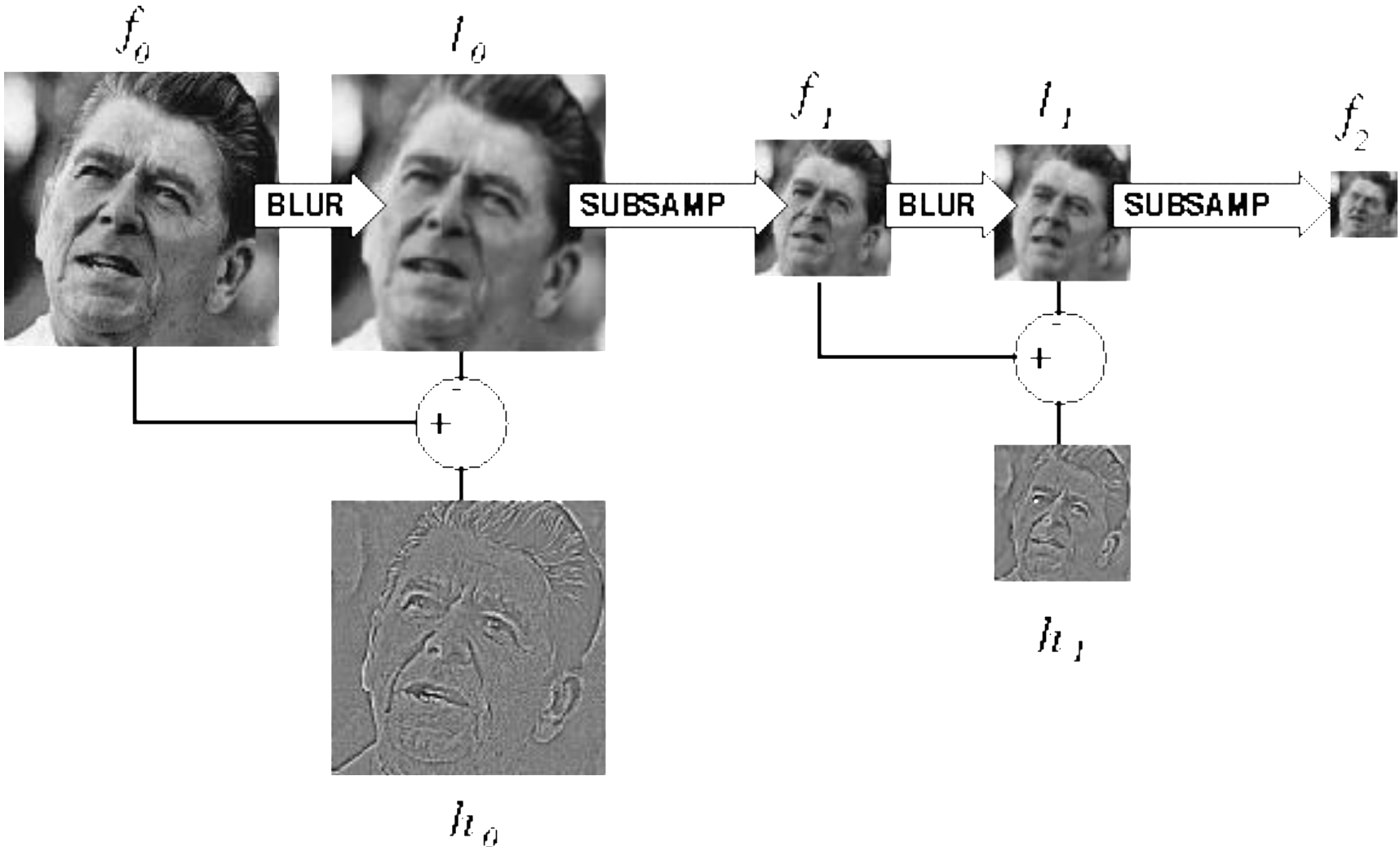


Laplacian



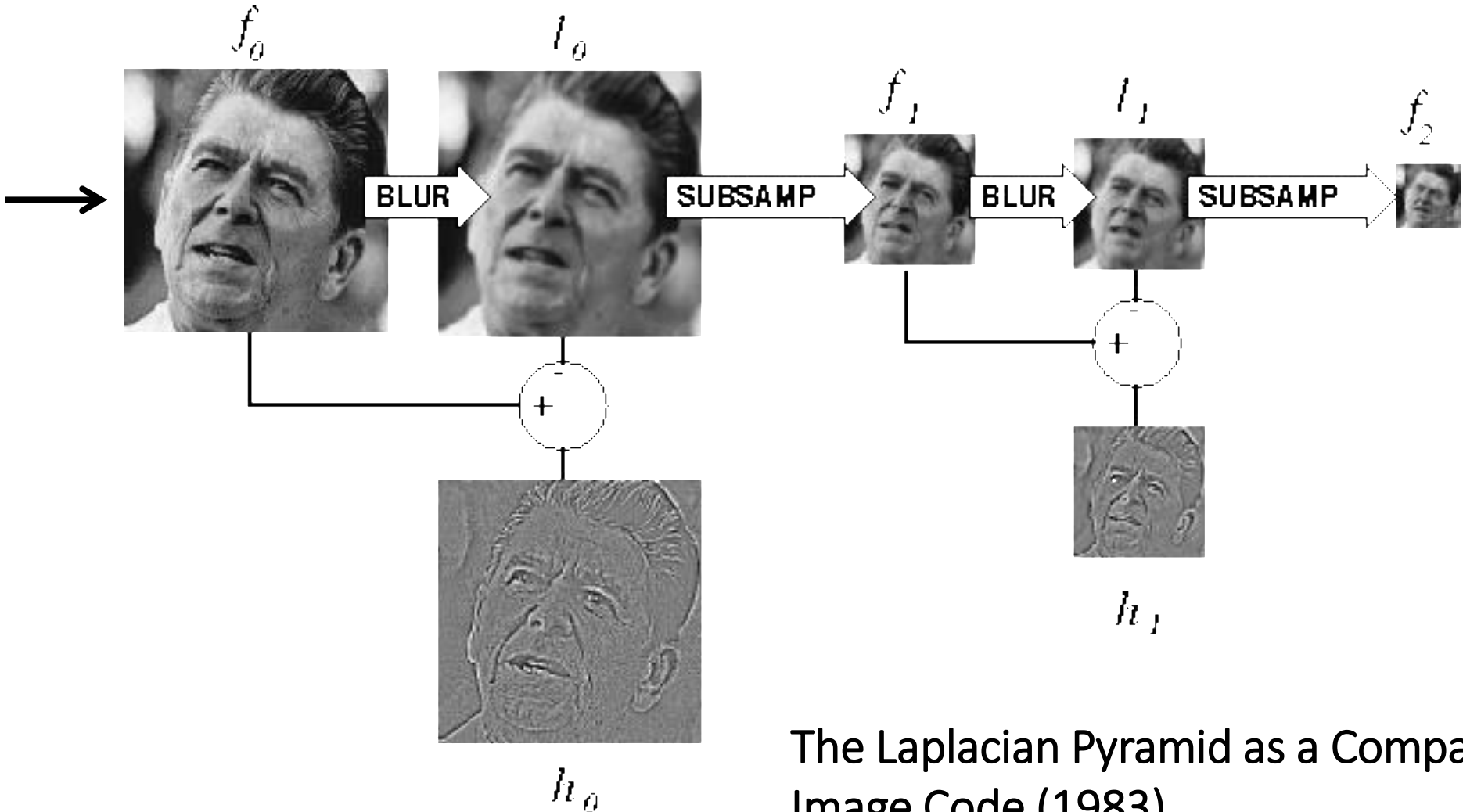
Difference of Gaussians approximates the Laplacian

# Why Reagan?



# Why Reagan?

Donald Reagan was President when the Laplacian pyramid was invented



The Laplacian Pyramid as a Compact Image Code (1983)

Peter J. Burt , Edward H. Adelson



Still used extensively



# Still used extensively



foreground details enhanced, background details reduced



input image



user-provided mask

# Still used extensively

Result from:

Paris et al., “Local Laplacian Filters: Edge-aware Image Processing with a Laplacian Pyramid,”  
SIGGRAPH 2011 and CACM 2015

Why “local”?

# Other types of pyramids

Steerable pyramid: At each level keep multiple versions, one for each direction.



Wavelets: Huge area in image processing (see 18-793).



# What are image pyramids used for?

image compression



multi-scale  
texture mapping

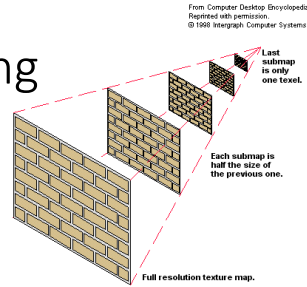
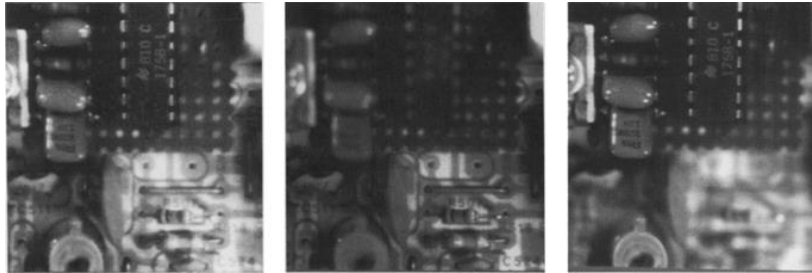


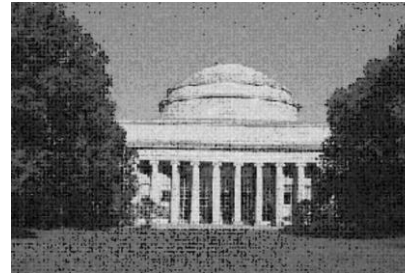
image blending



focal stack compositing



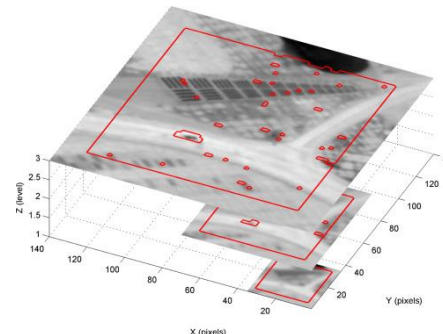
denoising



multi-scale detection



multi-scale registration



motion magnification



your homework 2

# References

Basic reading:

- Szeliski textbook, Sections 3.5

Additional reading:

- Burt and Adelson, “The Laplacian Pyramid as a Compact Image Code,” IEEE ToC 1983.  
the original Laplacian pyramid paper
- Paris et al., “Local Laplacian Filters: Edge-aware Image Processing with a Laplacian Pyramid,” SIGGRAPH 2011 and CACM 2015,  
great paper on modern uses of the Laplacian pyramid, see also the project website  
<https://people.csail.mit.edu/sparis/publi/2011/siggraph/>